ΔΙΑΦΟΡΙΚΟΣ ΚΑΙ ΟΛΟΚΛΗΡΩΤΙΚΟΣ ΛΟΓΙΣΜΟΣ I (1)

24 Ιανουαρίου 2014

ΣΧΟΛΗ Ν.Π.Δ

ΘΕΜΑ 1 (18 Μ)

$$y(x) = \left(1+\frac{r}{x}\right)^{k\cdot x}, \quad r, k \in \mathbb{R}, \quad k \neq 0$$

(α) $$y'(x) = k\left(1+\frac{r}{x}\right)^{kx-1}\left(-\frac{r}{x^2}\right) =$$

$$= -\frac{kr}{x^2}\left(1+\frac{r}{x}\right)^{kx-1}$$

(β) $$\lim_{x\to+\infty} y(x) = \lim_{x\to+\infty}\left(1+\frac{r}{x}\right)^{k\cdot x}$$

Θέτω $w = \frac{r}{x}$, όταν $x\to+\infty$ τότε $w\to 0$

$$= \lim_{\frac{r}{x}\to 0}\left[\left(1+\frac{r}{x}\right)^{\frac{x}{r}}\right]^{r\cdot k}$$

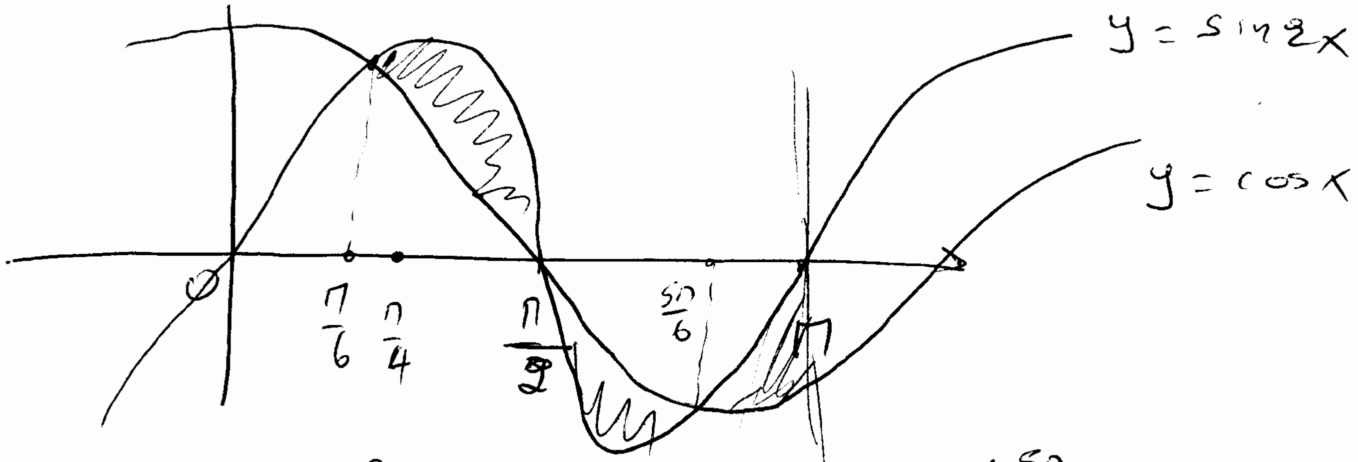
$$= \lim_{w\to 0}\left[\left(1+w\right)^{\frac{1}{w}}\right]^{r\cdot k}$$

$$= e^{k\cdot r}$$

ΘΕΜΑ 2ο:

(2)

Έκανα σχήμα Geogebra



$$A(x) = \underbrace{\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x - \cos x)\,dx}_{I_1} + \underbrace{\int_{\pi/2}^{\frac{5\pi}{6}} (\cos x - \sin 2x)\,dx}_{I_2}$$

$$+ \underbrace{\int_{\frac{5\pi}{6}}^{\pi} (\sin 2x - \cos x)\,dx}_{I_3}$$

$$I_1 = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\sin 2x - \cos x)\,dx = -\left[\frac{\cos 2x}{2}\right]_{\pi/6}^{\pi/2} - \left[\sin x\right]_{\pi/6}^{\pi/2}$$

$$= -\frac{\cos\pi}{2} + \frac{\cos\frac{\pi}{3}}{2} - \sin\frac{\pi}{2} + \sin\frac{\pi}{6}$$

$$= -\cancel{\frac{(-1)}{2}} + \frac{1}{4} - \cancel{1} + \cancel{\frac{1}{2}} = \frac{1}{4}$$

$$I_2 = \int_{\frac{\pi}{2}}^{\frac{5\pi}{6}} (\cos x - \sin 2x)\,dx$$

$$= \left[\sin x\right]_{\pi/2}^{5\pi/6} + \left[\frac{\cos 2x}{2}\right]_{\pi/2}^{5\pi/6}$$

$$= \sin\frac{5\pi}{6} - \sin\frac{\pi}{2} + \frac{\cos\frac{5\pi}{3}}{2} - \frac{\cos\pi}{2}$$

$$= \frac{1}{2} - 1 + \frac{1}{4} + \frac{1}{2} = \frac{1}{4}$$

$$I_3 = \int_{\frac{5\pi}{6}}^{\pi} (\sin 2x - \cos x)\,dx$$

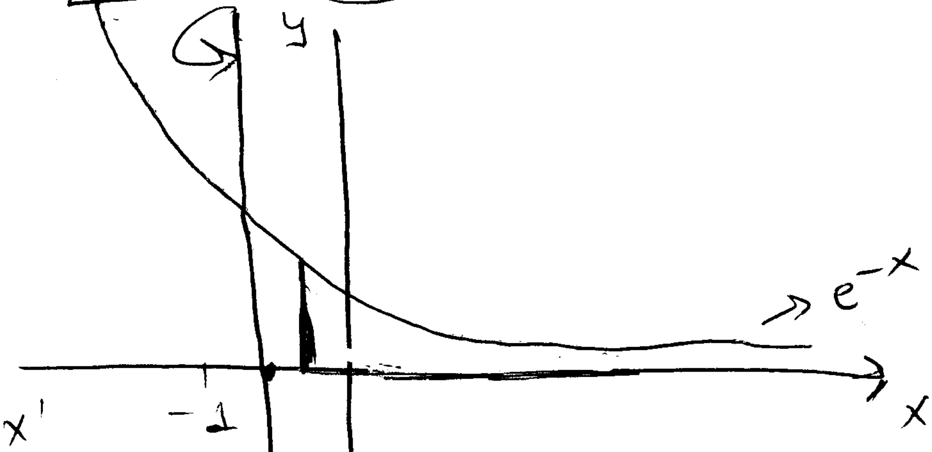
$$= -\left[\frac{\cos 2x}{2}\right]_{\frac{5\pi}{6}}^{\pi} - \left[\sin x\right]_{\frac{5\pi}{6}}^{\pi}$$

$$= -\frac{\cos 2\pi}{2} + \frac{\cos 5\pi/3}{2} - \sin\pi + \sin\frac{5\pi}{6}$$

$$= -\frac{1}{2} + \frac{1}{4} - 0 + \frac{1}{2} = \frac{1}{4}$$

Άρα $A(x) = I_1 + I_2 + I_3 = 0{,}75$

ΘΕΜΑ 3ο



$x = -\ln 2$ $\quad y'$

$$V(x) = \int_{-\ln 2}^{0} 2\pi(-\ln 2 - x)\cdot e^{-x}\,dx$$

$$= -2\pi \underbrace{\int_{-\ln 2}^{0} \ln 2\cdot e^{-x}\,dx}_{I_1} - 2\pi\cdot \underbrace{\int_{-\ln 2}^{0} x e^{-x}\,dx}_{I_2\,=\,-0{,}39}$$

$$I_1 = \ln 2 \int_{-\ln 2}^{0} e^{-x} = -\left[e^{-x}\right]_{-\ln 2}^{0} = (-e^{0} + e^{\ln 2})\ln 2 = \ln 2(-1+2)$$

$$= \ln 2$$

$$I_2 = \int_{-\ln 2}^{0} x(-e^{-x})'\,dx = \left[-xe^{-x}\right]_{-\ln 2}^{0} + \int_{-\ln 2}^{0} e^{-x}\,dx$$

$$= -0 + (-\ln 2\, e^{\ln 2}) + \left[-e^{-x}\right]_{-\ln 2}^{0}$$

$$= -\ln 2\cdot 2 + (-e^{0} + e^{\ln 2})$$

$$= -2\ln 2 - 1 + 2 = 1 - 2\ln 2 = -0{,}39$$

άρα $I = -2\pi\ln 2 - 2\pi(-0{,}39) = -2\cdot 3{,}14\cdot 0{,}69 - 2\cdot 3{,}14(-0{,}39)$

$= -6{,}28\cdot 0{,}69 + 6{,}28\cdot 0{,}39 = -0{,}3\cdot 6{,}28 = 1{,}884$ κ.μ

ΘΕΜΑ 4

$$\sec x \frac{dy}{dx} = e^{x-y} \Leftrightarrow \sec x \frac{dy}{dx} = \frac{e^x}{e^y}$$

$$\Leftrightarrow \frac{dy}{dx} e^{+y} = e^x \sec^{-1} x$$

$$\Rightarrow \int e^{+y} dy = \int e^x \sec^{-1} x \, dx$$

$$\Leftrightarrow e^y = e^x \sec^{-1} x - \int e^x \frac{1}{|x|\sqrt{x^2-1}} dx$$

ΘΕΜΑ 5

$$\frac{2x+2}{(x^2+1)(x-1)^2} = \frac{Ax+B}{x^2+1} + \frac{\Gamma}{x-1} + \frac{\Delta}{(x-1)^2}$$

$$\Leftrightarrow 2x+2 = (Ax+B)(x-1)^2 + \Gamma(x-1)(x^2+1) + \Delta(x^2+1)$$

$$\Leftrightarrow 2x+2 = (Ax+B)(x^2-2x+1) + \Gamma(x^3+x-x^2-1) + \Delta x^2 + \Delta$$

$$\Leftrightarrow 2x+2 = Ax^3 - 2x^2A + Ax + Bx^2 - 2xB + B + \Gamma x^3 + \Gamma x - \Gamma x^2 - \Gamma + \Delta x^2 + \Delta$$

$$\Leftrightarrow (A+\Gamma)x^3 + (-2A+B-\Gamma+\Delta)x^2 + (A-2B+\Gamma)x + (B-\Gamma+\Delta)$$

$$\begin{cases} A+\Gamma=0 \\ -2A+B-\Gamma+\Delta=0 \\ A-2B+\Gamma=2 \\ B-\Gamma+\Delta=2 \end{cases}$$