

Η Έννοια της Πιθανότητας

(1)

Ερωτήσεις Σωστό-Λάθος

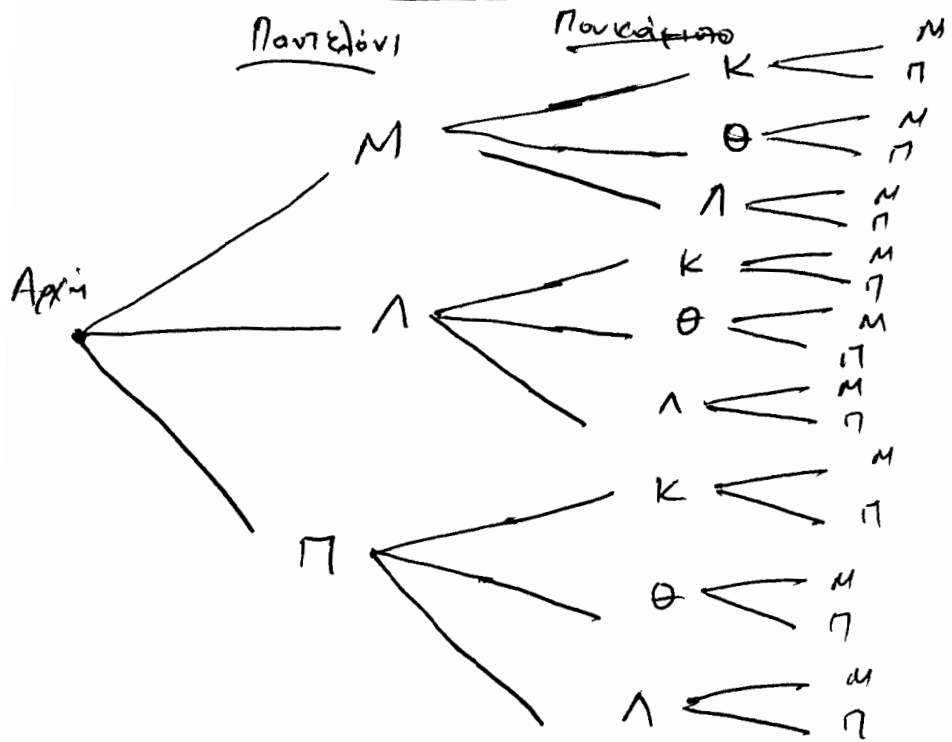
- (1) Σ (2) Σ (3) Σ (4) Σ (5) Λ (6) Σ (7) Σ (8) Λ
 (9) Λ (10) Λ (11) Σ (12) Σ (13) Σ (14) Λ (15) Σ (16) Λ (17) Σ
 (18) Λ (19) Λ (20) Σ

Ερωτήσεις Πολλαπλής Επιλογής

- (1) ε (2) ε (3) ε (4) Β (5) γ (6) Β (7) γ (8) δ (9) α

Ασκήσεις Αντιστοιχίας

Σημεία



$$\Omega = \{ \text{ΜΚΜ, ΜΚΠ, ΜΘΜ, ΜΘΠ, ΜΛΜ, ΜΛΠ, ΛΚΜ, ΛΚΠ, ΛΘΜ, ΛΘΠ, ΛΛΜ, ΛΛΠ, ΠΚΜ, ΠΚΠ, ΠΘΜ, ΠΘΠ, ΠΛΜ, ΠΛΠ} \}$$

(1) $P(M) = \frac{6}{18} = \frac{1}{3} \rightarrow \delta$ (3) $P(\Pi) = \frac{6}{18} = \frac{1}{3} \rightarrow \delta$

(2) $P(\text{OX}) = \frac{10}{18} = \frac{5}{9} \rightarrow \epsilon$ (4) $P(K) = 0$

(5) $P(\text{ΛΠΠΞ}) = \frac{3}{18} = \frac{1}{6} \rightarrow \text{Β}$

(--Λ--) = { ΜΛΜ, ΜΛΠ, ΛΛΜ, ΛΛΠ, ΠΛΜ, ΠΛΠ }
 (--Π) = { ΜΚΠ, ΜΘΠ, ΜΛΠ, ΛΚΠ, ΛΘΠ, ΛΛΠ, ΠΚΠ, ΠΘΠ, ΠΛΠ }

$$\text{δρα } (-A-) \cap (-B-) = \{M\Lambda\Gamma, \Lambda\Lambda\Gamma, \Pi\Lambda\Gamma\}$$

Ασκήσεις για λύση:

$$\textcircled{1} \quad P(A) = \frac{1}{3}, \quad P(B) = \frac{1}{4}$$

$$A \cap B \subseteq B \subseteq A \subseteq A \cup B$$

$$\text{δρα } A \subseteq A \cup B \Rightarrow P(A) \leq P(A \cup B)$$

$$\Leftrightarrow \boxed{\frac{1}{3} \leq P(A \cup B) \leq 1}$$

$$A \cap B \subseteq B \Rightarrow P(A \cap B) \leq P(B) \Leftrightarrow P(A \cap B) \leq \frac{1}{4}$$

$$\text{δρα } \boxed{0 \leq P(A \cap B) \leq \frac{1}{4}}$$

$$\textcircled{2} \quad \text{(I)} \quad \alpha) \quad P(A) = \frac{N(A)}{N(\Omega)} = \frac{4}{52} = \frac{1}{13}$$

$$\text{(B)} \quad P(\phi) = \frac{N(\phi)}{N(\Omega)} = \frac{12}{52} = \frac{3}{13}$$

$$\text{(γ)} \quad P(\Sigma) = \frac{N(\Sigma)}{N(\Omega)} = \frac{13}{52} = \frac{1}{4}$$

$$\text{(II)} \quad \alpha) \quad P(A) = \frac{N(A)}{N(\Omega)} = \frac{4}{51}$$

$$\text{(B)} \quad P(\phi) = \frac{11}{51} \quad \text{(γ)} \quad P(\Sigma) = \frac{12}{51}$$

$$(3) (a) P(A) = \frac{N(A)}{N(\Omega)} = \frac{38}{76} = \frac{1}{2}$$

(3)

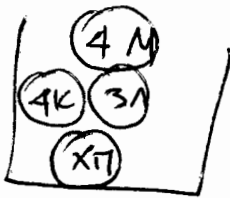
$$(b) P(K) = \frac{N(K)}{N(\Omega)} = \frac{21}{76} \quad \text{or} \quad P(K') = 1 - P(K) = 1 - \frac{21}{76} = \frac{55}{76}$$

$$(c) P(A \cap K) = \frac{N(A \cap K)}{N(\Omega)} = \frac{12}{76} = \frac{3}{19}$$

$$(d) P(A \cap M) = \frac{N(A \cap M)}{N(\Omega)} = \frac{12}{76} = \frac{3}{19}$$

$$(e) P(A \cap \Gamma) = \frac{N(A \cap \Gamma)}{N(\Omega)} = \frac{8}{76} = \frac{2}{19}$$

(4)



$$(a) P(A) = \frac{N(A)}{N(\Omega)} \Rightarrow \frac{1}{4} = \frac{4}{4+4+3+x} \quad (\Rightarrow)$$

$$\Rightarrow 11+x = 16 \Rightarrow \boxed{x=5}$$

$$(b) P(B) = P(A \cup \Gamma) = \frac{N(A \cup \Gamma)}{N(\Omega)} = \frac{1}{2} \quad (\Rightarrow) \frac{3+x}{4+4+3+x} = \frac{1}{2}$$

$$\Rightarrow 2 \cdot (3+x) = 11+x \Rightarrow 6+2x = 11+x$$

$$\Rightarrow \boxed{x=5}$$

$$(c) P(\bar{\Gamma}) = P(\Gamma') = 1 - P(\Gamma) = \frac{1}{2} \quad (\Rightarrow) P(\Gamma) = \frac{1}{2}$$

$$\Rightarrow \frac{N(\Gamma)}{N(\Omega)} = \frac{1}{2} \Rightarrow \frac{x}{4+4+3+x} = \frac{1}{2}$$

$$\Rightarrow 2x = 11+x \Rightarrow \boxed{x=11}$$

$$\textcircled{5} (a) P(K\Phi) = \frac{6}{52} = \frac{3}{26}$$

$$(B) P(10M) = \frac{N(10M)}{N(\Omega)} = \frac{2}{52} = \frac{1}{26}$$

$$(\gamma) P(M\pi) = \frac{N(M\pi)}{N(\Omega)} = \frac{10}{52} = \frac{5}{26}$$

$$\textcircled{6} P(H') = \frac{1}{3} \quad P(O') = \frac{1}{4} \quad P(H' \cap O') = \frac{1}{10}$$

$$P(H \cap O) = ?$$

$$P(H') = \frac{1}{3} \Leftrightarrow 1 - P(H) = \frac{1}{3} \Leftrightarrow P(H) = \frac{2}{3}$$

$$P(O') = \frac{1}{4} \Leftrightarrow 1 - P(O) = \frac{1}{4} \Leftrightarrow P(O) = \frac{3}{4}$$

$$P(H' \cap O') = P[(H \cup O)'] = \frac{1}{10} \Leftrightarrow P(H \cup O) = \frac{9}{10}$$

$$\text{d'apr} \quad P(H \cup O) = P(H) + P(O) - P(H \cap O)$$

$$\Leftrightarrow \frac{9}{10} = \frac{2}{3} + \frac{3}{4} - P(H \cap O)$$

$$\Leftrightarrow P(H \cap O) = \frac{8+9}{12} - \frac{9}{10} \Leftrightarrow P(H \cap O) = \frac{17}{12} - \frac{9}{10}$$

$$\Leftrightarrow P(H \cap O) = \frac{85 - 54}{60} = \frac{31}{60} = 0,5166$$

$$\textcircled{7} \quad P(B) = 2P(A) \quad , \quad P(\Gamma) = P(B) + P(A) \quad , \quad \boxed{P(A) = 0,1} \quad (5)$$

$$\boxed{P(B) = 0,2} \quad P(\Gamma) = 0,2 + 0,1 = 0,3 \quad \Leftrightarrow \quad \boxed{P(\Gamma) = 0,3}$$

$$P(A) + P(B) + P(\Gamma) + P(\Delta) = 1 \quad \Leftrightarrow \quad P(\Delta) = 1 - 0,1 - 0,3 - 0,2$$

$$\Leftrightarrow \quad \boxed{P(\Delta) = 0,4}$$

$$\textcircled{8} \quad P(A) = 2P(B) \quad \text{ka} \quad P(\Gamma) = 0,8P(B)$$

$$P(A) + P(B) + P(\Gamma) = 1 \quad \Leftrightarrow \quad 2P(B) + P(B) + 0,8P(B) = 1$$

$$\Leftrightarrow \quad 3,8P(B) = 1 \quad \Leftrightarrow \quad P(B) = \frac{10}{38} \quad \Leftrightarrow \quad \boxed{P(B) = \frac{5}{19}}$$

$$\text{de} \quad \boxed{P(A) = \frac{10}{19}} \quad \boxed{P(\Gamma) = \frac{4}{19}}$$

$$\textcircled{9} \quad P(A') = \frac{1}{2} \quad \Leftrightarrow \quad \boxed{P(A) = \frac{1}{2}}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \Leftrightarrow \quad \frac{3}{4} = \frac{1}{2} + P(B) - \frac{5}{12}$$

$$\Leftrightarrow \quad 9 = 6 + 12P(B) - 5 \quad \Leftrightarrow \quad 12P(B) = 8 \quad \Leftrightarrow \quad \boxed{P(B) = \frac{2}{3}}$$

$$P(A' \cup B') = P((A \cap B)') = 1 - P(A \cap B) = 1 - \frac{5}{12} = \frac{7}{12}$$

$$\text{de} \quad \boxed{P(A' \cup B') = \frac{7}{12}}$$

$$P(A \cap B') = P(A - B) = P(A) - P(A \cap B) = \frac{1}{2} - \frac{5}{12} = \frac{1}{12}$$

$$P(A' \cap B') = P((A \cup B)') = 1 - P(A \cup B) = 1 - \frac{3}{4} = \frac{1}{4}$$

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$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Leftrightarrow P(A \cup B) + P(A \cap B) = P(A) + P(B) \Leftrightarrow$$

$$\Leftrightarrow P(A \cup B) + P(A \cap B) > 1 \quad (1)$$

$0 \leq P(A \cup B) \leq 1$ Αν $P(A \cap B) = 0$ δίνονται να
τοξση ν οξέον (1)

11

$$P(A) = \frac{3}{5} \quad P(B) = \frac{4}{5}$$

$$(a) P(A) + P(B) = \frac{3}{5} + \frac{4}{5} = \frac{3+4}{5} = \frac{7}{5} > 1$$

Άρα $A \cap B \neq \emptyset$ αφού $P(A \cup B) \neq P(A) + P(B)$

$$(B) A \cap B \subseteq A \subseteq B \subseteq A \cup B$$

$$A \cap B \subseteq A \text{ άρα } P(A \cap B) \leq P(A) \Leftrightarrow P(A \cap B) \leq \frac{3}{5}$$

$$\Leftrightarrow 0 \leq P(A \cap B) \leq \frac{3}{5}$$

$$B \subseteq A \cup B \text{ άρα } P(B) \leq P(A \cup B) \Leftrightarrow \frac{4}{5} \leq P(A \cup B)$$

$$\text{Άρα } \frac{4}{5} \leq P(A \cup B) \leq 1$$

12

90 κάτοικοι

	A	OA	Σύνολο
H	25	35	60
OH	25	5	30
Σύνολο	50	40	90

$$(a) P(A \cup H) = P(A) + P(H) - P(A \cap H) = \frac{50}{90} + \frac{60}{90} - \frac{25}{90}$$

$$= \frac{85}{90} = \frac{17}{18}$$

$$(B) P(A' \cap H') = P[(A \cup H)'] = 1 - P(A \cup H) = \frac{1}{18}$$

(7)

$$(x) P[(A-H) \cup (H-A)] = P(A-H) + P(H-A) =$$

$$= P(A) - P(A \cap H) + P(H) - P(A \cap H) = P(A) + P(H) - 2P(A \cap H)$$

$$= \frac{50}{30} + \frac{60}{30} - 2 \cdot \frac{25}{30} = \frac{2}{3}$$

(13)

$$(a) P(A-B) = P(A \cup B) - P(B)$$

$$\Leftrightarrow P(A) - P(A \cap B) = P(A) + P(B) - P(A \cap B) - P(B)$$

10x24

$$(b) P(B-A) = P(A \cup B) - P(A)$$

$$\Leftrightarrow P(B) - P(A \cap B) = P(A) + P(B) - P(A \cap B) - P(A)$$

10x24

(14)

$$P(A) - P(B') \leq P(A \cap B) \Leftrightarrow$$

$$\Leftrightarrow P(A) - (1 - P(B)) \leq P(A \cap B) \Leftrightarrow P(A) - 1 + P(B) \leq P(A \cap B)$$

$$\Leftrightarrow P(A) + P(B) - P(A \cap B) \leq 1 \Leftrightarrow P(A \cup B) \leq 1$$

10x24

(15)

$$P(N) = \frac{1}{6} \quad P(M) = \frac{1}{5} \quad P(N \cap M) = \frac{1}{10}$$

$$(i) P(N \cup M) = P(N) + P(M) - P(N \cap M) = \frac{1}{6} + \frac{1}{5} - \frac{1}{10}$$

$$= \frac{5 + 6 - 3}{30} = \frac{8}{30} = \frac{4}{15}$$

$$(ii) P(N-M) = P(N) - P(N \cap M) = \frac{1}{6} - \frac{1}{10} = \frac{5-3}{30} = \frac{2}{30} = \frac{1}{15}$$

$$(iii) P[(N-M) \cup (M-N)] = P(N) + P(M) - 2P(N \cap M) =$$

$$= \frac{1}{6} + \frac{1}{5} - \frac{2}{10} = \frac{1}{6}$$

(8)

$$\textcircled{16} \quad P(A \cup B) = \frac{3}{4} \quad P(A \cap B) = \frac{1}{4} \quad P(A') = \frac{2}{3}$$

$$(i) \quad P(A) = 1 - P(A') \Leftrightarrow \boxed{P(A) = \frac{1}{3}}$$

$$(ii) \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Leftrightarrow \frac{3}{4} = \frac{1}{3} + P(B) - \frac{1}{4} \Leftrightarrow$$

$$\Leftrightarrow P(B) = 1 - \frac{1}{3} \Leftrightarrow \boxed{P(B) = \frac{2}{3}}$$

$$(iii) \quad P(A - B) = P(A) - P(A \cap B) = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$$

$$\textcircled{17} \quad P(A) = \frac{3}{4}, \quad P(B) = \frac{3}{8}$$

$$(i) \quad P(A) + P(B) = P(A \cup B) \text{ για να είναι ασυμβίβαστα}$$

$$P(A) + P(B) = \frac{6}{8} + \frac{3}{8} = \frac{9}{8} > 1 \text{ άρα ασυμβίβαστα.}$$

$$(ii) \quad \frac{3}{4} \leq P(A \cup B) \leq 1 \quad A \subseteq A \cup B \Rightarrow P(A) \leq P(A \cup B)$$

$$\Leftrightarrow \frac{3}{4} \leq P(A \cup B) \text{ και } \frac{3}{4} \leq P(A \cup B) \leq 1$$

Αφού
είναι
πίθανότητα

$$(iii) \quad \frac{1}{8} \leq P(A \cap B) \leq \frac{3}{8}$$

$$A \cap B \subseteq B \Rightarrow P(A \cap B) \leq P(B) \Leftrightarrow P(A \cap B) \leq \frac{3}{8} \quad (i)$$

$$\frac{1}{8} \leq P(A \cap B) \Leftrightarrow \frac{1}{8} \leq P(A) + P(B) - P(A \cup B)$$

$$\Leftrightarrow \frac{1}{8} \leq \frac{3}{4} + \frac{3}{8} - P(A \cup B) \Leftrightarrow P(A \cup B) \leq \frac{3}{4} + \frac{2}{8}$$

$$\Leftrightarrow P(A \cup B) \leq 1 \text{ ισχύει}$$

$$(18) \quad P(A) = \frac{5}{6} \quad P(B) = \frac{3}{4} \quad (9)$$

$$P(A' \cap B') = \frac{5}{8} \Rightarrow P(A \cup B)' = \frac{5}{8} \Rightarrow P(A \cup B) = \frac{3}{8}$$

Εξω $\frac{3}{4} > \frac{3}{8}$ άρα δεν υπάρχουν τέτοια ενδεχ.

$$\text{άρα } B \subseteq A \cup B \text{ άρα } P(B) \leq P(A \cup B)$$

$$(19) \quad P(B) = \frac{7}{15} \quad P(A \cap B) = \frac{2}{15}$$

$$P(A - B) = \frac{2}{15} \Rightarrow P(A) - P(A \cap B) = \frac{2}{15} \quad (1)$$

$$\text{άρα } P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\stackrel{(1)}{=} \frac{2}{15} + \frac{7}{15} = \frac{9}{15} \text{ άρα } P(A \cup B) = \frac{9}{15} = \frac{3}{5}$$

$$(20) \quad (a) \quad P(A \cap B) + P(A) - P(A \cap B) \leq 1 \Rightarrow P(A) \leq 1 \text{ (οξση)}$$

$$(b) \quad [P(A \cap B)]^2 - [P(A - B)]^2 \leq 1 - 2P(A - B)$$

$$\Rightarrow [P(A \cap B)]^2 \leq [P(A - B)]^2 - 2P(A - B) + 1$$

$$\Rightarrow [P(A \cap B)]^2 \leq [P(A - B) - 1]^2$$

$$\Rightarrow [P(A \cap B)]^2 - [P(A - B) - 1]^2 \leq 0$$

$$\Rightarrow \underbrace{[P(A \cap B) - P(A - B) + 1]}_{\leq 0 \text{ άρα } 1 = \text{επιωθηα}} \underbrace{[P(A \cap B) + P(A - B) - 1]}_{\geq 0} \leq 0$$

$$P(A \cap B) - P(A) + P(A \cap B) + 1 = 2P(A \cap B) - P(A) + 1$$

$$0 \leq P(A \cap B) \leq 1 \Rightarrow 0 \leq 2P(A \cap B) \leq 2 \quad (1)$$

$$0 \leq P(A) \leq 1 \Rightarrow -1 \leq -P(A) \leq 0 \Rightarrow 0 \leq 1 - P(A) \leq 1 \quad (2)$$

$$\text{άρα } (1) + (2) \quad \boxed{0 \leq 2P(A \cap B) - P(A) + 1 \leq 3}$$

21

1Eσw
 $P(A) = \frac{1}{5} = 0,2$ $P(B) = \frac{1}{8} = 0,125$
 $P(A \cup B) = P(A) + P(B) = 0,325$

$2P^2(A) + 2P^2(B) > 2P(A \cup B) - 1$

A' + ε) >

$2 \cdot 0,2^2 + 2 \cdot 0,125^2 = 0,08 + 0,03125 = 0,11125$

B' + ε) >

$2 \cdot 0,325 - 1 = 0,35$ & Tono !!!

22

$\frac{P(A)}{P(B)} + \frac{P(B)}{P(A)} \geq 2$ (⇐) $P(A) \cdot P(B) > 0$

$P(A)^2 + P(B)^2 \geq 2P(A) \cdot P(B)$ (⇐) $(P(A) - P(B))^2 \geq 0$ 10x0E4

23

$\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ $A = \{\omega_1, \omega_2\}$, $B = \{\omega_2, \omega_3\}$

$P(A) = \frac{1}{4}$ (⇐) $P(\omega_1) + P(\omega_2) = \frac{1}{4}$

$P(\omega_2) + P(\omega_3) = \frac{1}{8}$ ka) $P(\omega_2) = \frac{1}{12}$

Apa $P(\omega_3) = \frac{1}{8} - \frac{1}{12}$ ⇐ $P(\omega_3) = \frac{3-2}{24} = \frac{1}{24}$

de $P(\omega_3) = \frac{1}{24}$ $P(\omega_1) = \frac{1}{4} - \frac{1}{12}$ ⇐ $P(\omega_1) = \frac{3-1}{12}$

⇐ $P(\omega_1) = \frac{1}{6}$

$P(\omega_1) + P(\omega_2) + P(\omega_3) + P(\omega_4) = 1$ (⇐)

⇐ $P(\omega_4) = 1 - \frac{1}{6} - \frac{1}{12} - \frac{1}{24} = \frac{24-4-2-1}{24}$

⇐ $P(\omega_4) = \frac{17}{24}$

24

$$P(A) = P(\omega_3) \Leftrightarrow$$

$$P(\omega_1) + P(\omega_2) = P(\omega_3) \quad (1)$$

(11)

$$P(B) = 10 P(\omega_1) \Leftrightarrow$$

$$P(\omega_2) + P(\omega_3) + P(\omega_4) = 10 P(\omega_1) \quad (2)$$

$$P(A' \cap B) = \frac{8}{11}$$

$$A' = \{\omega_3, \omega_4\}, \quad B = \{\omega_2, \omega_3, \omega_4\}$$

deci

$$A' \cap B = \{\omega_3, \omega_4\} \text{ deci}$$

$$P(\omega_3) + P(\omega_4) = \frac{8}{11} \quad (3)$$

$$\text{Apoi din } (2) \stackrel{(3)}{\Rightarrow} P(\omega_2) + \frac{8}{11} = 10 P(\omega_1)$$

$$\text{deci } 10 P(\omega_1) - P(\omega_2) = \frac{8}{11} \quad (4)$$

$$P(\omega_1) + P(\omega_2) + P(\omega_3) + P(\omega_4) = 1 \stackrel{(1)}{=} P(\omega_3) + P(\omega_3) + P(\omega_4) = 1$$

$$\text{deci } 2 P(\omega_3) + P(\omega_4) = 1 \quad (5)$$

Acum sa luimera sa (3), (5)

$$P(\omega_3) = 1 - \frac{8}{11} \Rightarrow$$

$$P(\omega_3) = \frac{3}{11}$$

$$\text{Apoi } P(\omega_4) = \frac{8}{11} - \frac{3}{11} \Rightarrow$$

$$P(\omega_4) = \frac{5}{11}$$

$$\text{Apoi din } (1) \Rightarrow P(\omega_1) + P(\omega_2) = \frac{3}{11} \text{ ca } 10 P(\omega_1) - P(\omega_2) = \frac{8}{11}$$

$$11 P(\omega_1) = 1 \Rightarrow$$

$$P(\omega_1) = \frac{1}{11}$$

deci

$$P(\omega_2) = \frac{2}{11}$$