

Λύσεις του 2^{ου} ΘΕΜΑΤΟΣ

① (a) $w = 2, a_1 = 1$

$$a_{100} = a_1 + 99 \cdot w = 1 + 99 \cdot 2 = 1 + 198 = 199$$

(B)
$$S_n = \frac{v}{2} [2 \cdot a_1 + (n-1) \cdot 2] = \frac{v}{2} (2 + 2n - 2)$$

$$= v \cdot 2$$

② (a) $D_f = \mathbb{R} - \{3\}$

(B) $f(x) = \frac{(x-2)(x-3)}{x-3} = x-2$

(γ) • Άξονα x'x θέτουμε $f(x) = 0 \Rightarrow x = 2$ απεχ $A(2, 0)$
 • Άξονα y'y " $x = 0$ απεχ $B(0, -2)$

③ (a) Πρέπει $\Delta \geq 0 \quad a = 1, B = -2, \gamma = \lambda^2 + 2 - 1$

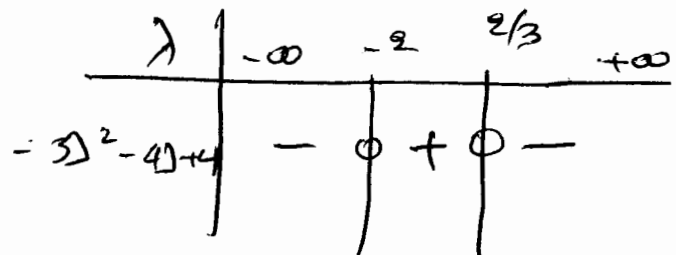
$$\Delta = B^2 - 4a\gamma = 2^2 - 4 \cdot 1 (\lambda^2 + 2 - 1) = 4 - 4\lambda^2 - 4\lambda + 4$$

$$= -3\lambda^2 - 4\lambda + 4$$

Θέτουμε $\Delta \geq 0 \Leftrightarrow -3\lambda^2 - 4\lambda + 4 \geq 0 \quad \Delta' = 16 - 4(-3) \cdot 4$

$$= 16 + 48 = 64$$

$$\lambda_{1,2} = \frac{4 \pm 8}{-6} \quad \begin{matrix} -2 \\ \frac{2}{3} \end{matrix}$$



$$\lambda \in [-2, \frac{2}{3}]$$

$$(B) \quad S = -\frac{B}{a} = \lambda, \quad P = \lambda^2 + \lambda - 1$$

(2)

$$\lambda^2 - (\lambda^2 + \lambda - 1) \geq 0 \Leftrightarrow -\lambda + 1 \geq 0 \Leftrightarrow \boxed{\lambda \leq 1}$$

(4) (a) (i) Πραγματοποιείται το A ή το B

(ii) " " συγχρόως τα B και Γ

(iii) " " " τα A και B και Γ

(iv) Δεν πραγματοποιείται το A

$$(B) \quad (i) \quad N(A) = 5, \quad N(B) = 5, \quad N(\Gamma) = 5, \quad N(\Omega) = 14$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{5}{14} + \frac{5}{14} - \frac{2}{14} = \frac{8}{14}$$

$$(ii) \quad P(B \cap \Gamma) = \frac{N(B \cap \Gamma)}{N(\Omega)} = \frac{2}{14}$$

$$(iii) \quad P(A \cap B \cap \Gamma) = \frac{N(A \cap B \cap \Gamma)}{N(\Omega)} = \frac{1}{14}$$

$$(iv) \quad P(A') = 1 - P(A) = 1 - \frac{5}{14} = \frac{9}{14}$$

$$(5) \quad \alpha_7 = 36 \quad S_{10} = 300$$

$$\alpha_1 + 6\omega = 36 \Leftrightarrow \alpha_1 = 36 - 6\omega = 6(6 - \omega)$$

$$\begin{aligned} S_{10} &= \frac{10}{2} [2 \cdot \alpha_1 + 9 \cdot \omega] = 5 [2 \cdot 6 \cdot (6 - \omega) + 9\omega] \\ &= 5 (72 - 12\omega + 9\omega) = 5 (72 - 3\omega) \end{aligned}$$

$$\text{οπότε} \quad 5 \cdot 72 - 15\omega = 300 \Leftrightarrow 15\omega = 360 - 300$$

$$\Leftrightarrow 15\omega = 60 \Leftrightarrow \boxed{\omega = 4} \quad \text{και} \quad \alpha_1 = 36 - 24 = 12$$

$$\text{οπότε} \quad \boxed{\alpha = 4}$$

$$\begin{array}{cccccccc}
 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
 12 & \xrightarrow{+4} & 16 & \rightarrow & 20 & \rightarrow & 24 & \rightarrow & 28 & \rightarrow & 32 & \rightarrow & 36 & \rightarrow & 40 \\
 & & 9 & & 10 & & & & & & & & & & & \\
 & & \rightarrow & 44 & \rightarrow & 48 & & & & & & & & & &
 \end{array}
 \quad (3)$$

6 $x^2 - 2\lambda x + 4(\lambda - 1) = 0$, $a = -1$, $B = -2\lambda$, $\gamma = 4(\lambda - 1)$

(a) $\Delta = (-2\lambda)^2 - 4 \cdot 1 \cdot 4(\lambda - 1) = 4\lambda^2 - 16\lambda + 16 = 4(\lambda^2 - 4\lambda + 4)$

(B) $\Delta \geq 0 \Rightarrow 4(\lambda - 2)^2 \geq 0$

(d) $S = x_1 + x_2 = -\frac{B}{a} = 2\lambda$, $P = x_1 \cdot x_2 = \frac{\gamma}{a} = 4(\lambda - 1)$

$\theta \in]0, \pi[\Rightarrow 2\lambda = 4\lambda - 4 \Leftrightarrow 2\lambda = 4 \Leftrightarrow \boxed{\lambda = 2}$

7 $x^2 + 2\lambda x + \lambda - 2 = 0$ $a = 1$, $B = 2\lambda$, $\gamma = \lambda - 2$

(a) $\Delta = 4\lambda^2 - 4\lambda + 8 = 4(\lambda^2 - \lambda + 2)$

(B) $\Delta' = 1 - 8 = -7 < 0 \Rightarrow \Delta > 0$

(d) $x_1 + x_2 = -x_1 \cdot x_2 \Leftrightarrow -2\lambda = -(\lambda - 2)$

$\Leftrightarrow -2\lambda = -\lambda + 2 \Leftrightarrow \boxed{\lambda = -2}$

8 (a) $|2x - 1| = 3 \Leftrightarrow 2x - 1 = 3 \vee 2x - 1 = -3$

$\Leftrightarrow 2x = 4 \vee 2x = -2 \Leftrightarrow \boxed{x = 2} \vee \boxed{x = -1}$

(B) $a < B \Rightarrow a = -1 \vee a = 2$

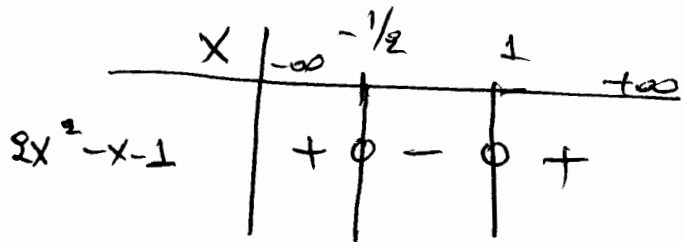
$-x^2 + 2x + 3 = 0 \Rightarrow \Delta = 4 - 4 \cdot 1 \cdot (-3) = 4 + 12 = 16$

$x_{1,2} = \frac{-2 \pm 4}{-2} = \begin{array}{l} \rightarrow -1 \\ \searrow 3 \end{array}$

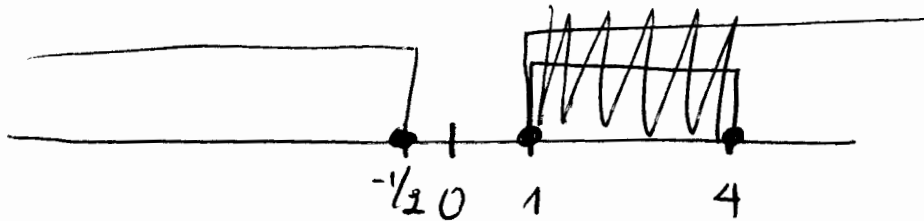
9 (a) $|2x-5| \leq 3$ και $2x^2 - x - 1 \geq 0$

$$-3 \leq 2x-5 \leq 3 \Leftrightarrow 2 \leq 2x \leq 8 \Leftrightarrow 1 \leq x \leq 4$$

$$\Delta = 1 - 4 \cdot 2(-1) = 9 \quad x_{1,2} = \frac{1 \pm 3}{4} \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$$



$$x \in (-\infty, -\frac{1}{2}] \cup [1, +\infty)$$



$$\text{Τελικά } x \in [1, 4]$$

10 (a) $\lambda \cdot x = x + \lambda^2 - 1$

$$\Leftrightarrow (\lambda - 1)x = (\lambda - 1)(\lambda + 1)$$

(b) Για $\lambda \neq 1$ $x = \frac{(\lambda - 1)(\lambda + 1)}{\lambda - 1} \Leftrightarrow x = \lambda + 1$

(γ) Πρέπει $\lambda - 1 = 0$ και $(\lambda - 1)(\lambda + 1) = 0$

$$\Leftrightarrow \lambda = 1 \quad \text{και} \quad \lambda = 1 \vee \lambda = -1$$

$$\text{Τελικά } \boxed{\lambda = 1}$$

11 $0 < a < 1$

(a) $a^3 < a \Leftrightarrow a^3 - a < 0 \Leftrightarrow a(a^2 - 1) < 0$

$$\Leftrightarrow a(a-1)(a+1) < 0$$

+ - + 1 < x < 1

$$a < 1 \Leftrightarrow a - 1 < 0$$

$$(B) \quad 0 < a < 1, \quad 0, \quad a^3, \quad 1, \quad a, \quad \frac{1}{a}$$

$$0 > \frac{1}{a} > 1 \quad \text{or} \quad 0 < a^3 < a < 1 < \frac{1}{a}$$

$$(12) (a) \quad (x-1)^2 + (y+3)^2 = x^2 + y^2 - 2x + 6y + 10$$

$$(B) \quad (x-1)^2 + (y+3)^2 = 0 \Leftrightarrow x=1 \text{ and } y=-3$$

$$(13) (a) \quad D_f = \mathbb{R} \setminus \{-1, 1\}$$

$$(B) \quad \Delta = 25 - 4 \cdot 2 \cdot 3 = 1 \quad x_{1,2} = \frac{5 \pm 1}{4} \begin{cases} \frac{3}{2} \\ 1 \end{cases}$$

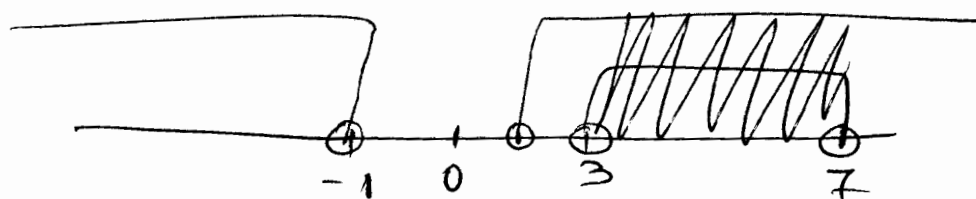
$$2(x-1)\left(x-\frac{3}{2}\right) = (x-1)(2x-3)$$

$$(g) \quad f(x) = \frac{(x-1)(2x-3)}{(x-1)(x+1)} = \frac{2x-3}{x+1}$$

$$(14) (a) \quad |x-5| < 2 \Leftrightarrow -2 < x-5 < 2 \Leftrightarrow 3 < x < 7$$

$$(B) \quad 2-3x > 5 \quad \text{and} \quad 2-3x < -5 \Leftrightarrow 3x < -3 \Leftrightarrow x < -1$$

$$\text{and} \quad 3x > 7 \Leftrightarrow x > \frac{7}{3}$$



$$x \in (3, 7)$$

15 $2x^2 - 3x + 1$
 (a) $\Delta = (-3)^2 - 4 \cdot 2 \cdot 1 = 9 - 8 = 1$, $x_{1,2} = \frac{3 \pm 1}{4} \begin{matrix} 1 \\ \frac{1}{2} \end{matrix}$

(b) $x \begin{matrix} | -\infty & \frac{1}{2} & 1 & +\infty \\ \hline 2x^2 - 3x + 1 & + & - & + \end{matrix} \quad x \in \left(\frac{1}{2}, 1 \right)$

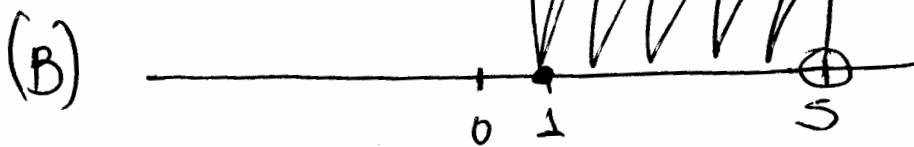
(c) $\frac{\sqrt{3}}{2} \in \left(\frac{1}{2}, 1 \right)$ and $\frac{1}{\sqrt{2}} \in \left(\frac{1}{2}, 1 \right)$

16 (a) $3x - 1 < x + 9 \Rightarrow 3x - x < 1 + 9$
 $\Rightarrow 2x < 10 \Rightarrow \boxed{x < 5}$

$2 - \frac{x}{2} \leq x + \frac{1}{2} \Rightarrow 4 - x \leq 2x + 1$

$\Rightarrow -x - 2x \leq -4 + 1 \Rightarrow -3x \leq -3$

$\Rightarrow \boxed{x \geq 1}$



$1 \leq x < 5$

17 $f(x) = x^2 + 2x - 15$

(a) $f(-1) = (-1)^2 + 2(-1) - 15 = 1 - 2 - 15 = -16$

$f(0) = 0^2 + 2 \cdot 0 - 15 = -15$

$f(1) = 1^2 + 2 \cdot 1 - 15 = 1 + 2 - 15 = -13$

$f(-1) + f(0) + f(1) = -16 - 15 - 13 = -44$

(B) • Aζουα x'x, θέτουμε $f(x) = 0$

(7)

$(\Rightarrow) x^2 + 2x - 15 = 0 \quad \Delta = 4 - 4 \cdot 1 \cdot (-15) = 4 + 60 = 64$

$x_{1,2} = \frac{-2 \pm 8}{2} \begin{matrix} 3 \\ -5 \end{matrix}$

A(3,0), B(-5,0)

• Aζουα y'y: θέτουμε $x=0$

$f(x) = -15 \quad \Gamma(0, -15)$

18

(a) $|x-2| = \sqrt{3} \quad (\Rightarrow) x-2 = \sqrt{3} \quad \text{ή} \quad x-2 = -\sqrt{3}$

$(\Rightarrow) x = 2 + \sqrt{3} \quad \text{ή} \quad x = 2 - \sqrt{3}$

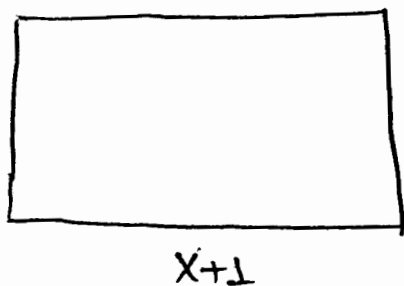
(B) $x^2 - 5x + p = 0$

$S = x_1 + x_2 = 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

$P = x_1 \cdot x_2 = (2 - \sqrt{3})(2 + \sqrt{3}) = 2^2 - (\sqrt{3})^2 = 4 - 3 = 1$

$x^2 - 4x + 1 = 0$

19



$x > 0$

(a) $\Pi = 2x + 2(x+1)$
 $= 2x + 2x + 2$
 $= 4x + 2$

$E = x(x+1) = x^2 + x$

(B) $E = 90 \quad (\Rightarrow) x^2 + x = 90 \quad (\Rightarrow) x^2 + x - 90 = 0$
 $\Delta = 1 - 4 \cdot 1 \cdot (-90) = 361, \quad x_{1,2} = \frac{-1 \pm 19}{2} \begin{matrix} 9 \\ -10 \text{ ανοπ.} \end{matrix}$

Διαστάσεις: 9×10

(8)

20 (a) $a_3 = a_1 \cdot \lambda^{3-1} = 1 \Rightarrow a_1 \cdot \lambda^2 = 1$ (1)

$$a_5 = a_1 \cdot \lambda^{5-1} = 4 \Rightarrow a_1 \cdot \lambda^4 = 4$$
 (2)

(2) \div (1) $\frac{a_1 \cdot \lambda^4}{a_1 \cdot \lambda^2} = \frac{4}{1} \Rightarrow \lambda^2 = 4 \Rightarrow \boxed{\lambda = 2}$ $\lambda = -2$

4 (1) $\Rightarrow a_1 \cdot 2^2 = 1 \Rightarrow 4 \cdot a_1 = 1 \Rightarrow \boxed{a_1 = \frac{1}{4}}$

(B) $a_v = a_1 \cdot \lambda^{v-1} = \frac{1}{4} \cdot 2^{v-1} = \frac{2^{v-1}}{4} = \frac{2^{v-1}}{2^2} = 2^{v-1-2} = 2^{v-3} \Rightarrow \boxed{a_v = 2^{v-3}}$

21 $x^2 + 2\lambda x + 4(\lambda-1) = 0$, $a=1$, $B=2\lambda$, $\gamma=4(\lambda-1)$

(a) $\Delta = B^2 - 4a\gamma = 4\lambda^2 - 4 \cdot 1 \cdot 4(\lambda-1) = 4\lambda^2 - 16(\lambda-1)$
 $= 4\lambda^2 - 16\lambda + 16 = 4(\lambda^2 - 4\lambda + 4) = 4(\lambda-2)^2 \geq 0$

(B) Αφού το $\Delta \geq 0$ ισχύει

(γ) $S = x_1 + x_2 = -\frac{B}{a} = -\frac{2\lambda}{1} = -2\lambda$

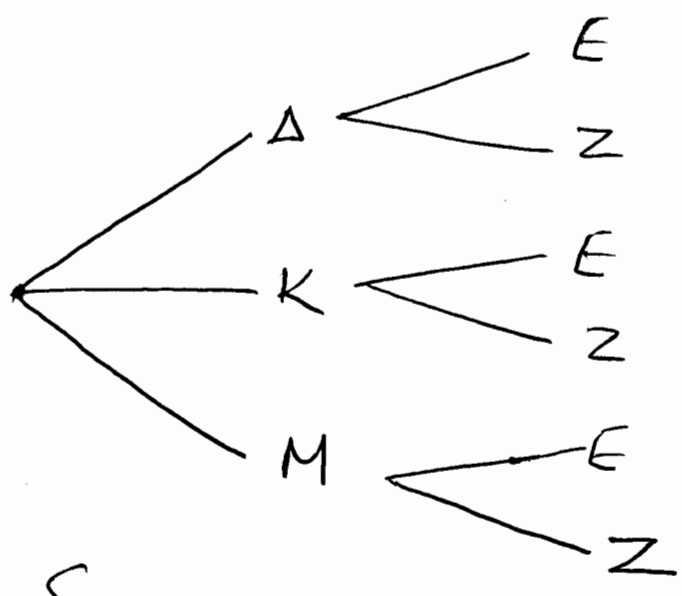
$$P = x_1 \cdot x_2 = \frac{\gamma}{a} = 4\lambda - 4$$

$$(-2\lambda)^2 + 4\lambda - 4 + 5 = 0 \Rightarrow 4\lambda^2 + 4\lambda + 1 = 0$$

$$\Rightarrow (2\lambda + 1)^2 = 0 \Rightarrow 2\lambda = -1 \Rightarrow \lambda = -\frac{1}{2}$$

22

(a)



$$\underline{\Omega} = \{ \Delta E, \Delta Z, KE, KZ, ME, MZ \}, N(\underline{\Omega}) = 6$$

$$(B) P(A) = \frac{N(A)}{N(\underline{\Omega})} = \frac{4}{6} = \frac{2}{3}$$

$$P(B) = \frac{N(B)}{N(\underline{\Omega})} = \frac{3}{6} = \frac{1}{2}$$

$$P(\Gamma) = \frac{N(\Gamma)}{N(\underline{\Omega})} = \frac{2}{6} = \frac{1}{3}$$

23

$$(a) \frac{|x+1|}{3} - \frac{|x+1|+4}{5} = \frac{2}{3}$$

$$\Leftrightarrow 5|x+1| - 3(|x+1|+4) = 5 \cdot 2$$

$$\Leftrightarrow 5|x+1| - 3|x+1| - 12 = 10$$

$$\Leftrightarrow 2|x+1| = 22 \Leftrightarrow |x+1| = 11 \Leftrightarrow \begin{matrix} x+1 = 11 \\ \vee \\ x+1 = -11 \end{matrix}$$

$$\Leftrightarrow x = 10 \vee x = -12$$

$$(B) -x^2 + 2x + 3 \leq 0 \quad \Delta = 4 - 4 \cdot (-1) \cdot 3 = 4 + 12 = 16$$

$$x_{1,2} = \frac{-2 \pm 4}{-2} \quad \begin{array}{c} -1 \\ 3 \end{array} \quad \begin{array}{c} x \\ -x^2 + 2x + 3 \end{array} \begin{array}{c} | \\ | \\ | \end{array} \begin{array}{c} -\infty \\ -1 \\ 3 \\ +\infty \end{array} \begin{array}{c} - \\ + \\ - \end{array}$$

$$x \in (-\infty, -1] \cup [3, +\infty)$$

$$-12, 10 \in (-\infty, -1] \cup [3, +\infty) \text{ \u03c1\u03c1\u03b1 \u03b5\u03c1\u03c1\u03b1 \u03b7 \u03b8\u03b5\u03c1\u03c9\u03c9}$$

24

(a) (i) 0 \u03c1\u03b1\u03b8\u03bc\u03b7\u03c3 \u03c3\u03c5\u03c6\u03b5\u03c1\u03b5\u03c7\u03b5\u03b9 \u03c3\u03bc\u03bd \u03b8.0 \u2193 \u03c3\u03bc\u03bd 0.7

(ii) 0 " " " \u03b8.0 \u03ba\u03b9 " "

(iii) 0 \u03c1\u03b1\u03b8\u03bc\u03b7\u03c3 " \u03c3\u03bc\u03bd 0.7 \u03ba\u03b9 \u03cc\u03c7\u03b9 \u03c3\u03bc\u03bd \u03b8.0

(iv) 0 \u03c1\u03b1\u03b8\u03bc\u03b7\u03c3 \u0394\u03b5\u03c9 \u03c3\u03c5\u03c6\u03b5\u03c1\u03b5\u03c7\u03b5\u03b9 \u03c3\u03bc\u03bd \u03b8.0

$$(B) (i) P(B-A) = P(B) - P(A \cap B) = \frac{30}{100} - \frac{15}{100} = 15\%$$

$$(ii) P(A' \cap B') = P[(A \cup B)'] = 1 - P(A \cup B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{25}{100} + \frac{30}{100} - \frac{15}{100} \\ = \frac{40}{100}$$

$$\text{\u03c1\u03c1\u03b1 } P(A' \cap B') = 1 - \frac{40}{100} = 60\%$$

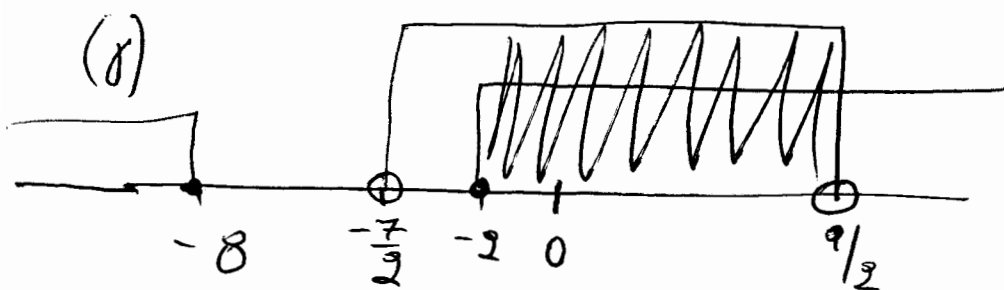
$$\textcircled{25} \text{ (a) } \left| x - \frac{1}{2} \right| < 4 \Leftrightarrow -4 < x - \frac{1}{2} < 4 \quad (11)$$

$$\Leftrightarrow -4 + \frac{1}{2} < x < 4 + \frac{1}{2} \Leftrightarrow -\frac{8}{2} + \frac{1}{2} < x < \frac{8}{2} + \frac{1}{2}$$

$$\Leftrightarrow -\frac{7}{2} < x < \frac{9}{2}$$

$$\text{(B) } |x+5| \geq 3 \Leftrightarrow x+5 \geq 3 \vee x+5 \leq -3$$

$$\Leftrightarrow x \geq -2 \vee x \leq -8$$



$$x \in \left[-2, \frac{9}{2} \right)$$

$$\textcircled{26} \text{ (a) } \forall \underline{a < 0} \text{ τότε } a + \frac{1}{a} \leq -2$$

$$\Leftrightarrow a + \frac{1}{a} + 2 \leq 0 \Leftrightarrow \frac{a^2 + 1 + 2a}{a} \leq 0$$

$$\Leftrightarrow \frac{a^2 + 2a + 1}{a} \leq 0 \Leftrightarrow \frac{(a+1)^2}{a} \leq 0 \quad \text{10x5ε}$$

$$\alpha \neq a \quad (a+1)^2 \geq 0 \quad \forall a, a < 0$$

$$\text{(B) } |a| + \frac{1}{|a|} \geq 2 \quad |a| > 0 \Leftrightarrow |a|^2 + 1 \geq 2|a|$$

$$\Leftrightarrow |a|^2 - 2|a| + 1 \geq 0 \Leftrightarrow (|a| - 1)^2 \geq 0 \quad \text{10x5ε}$$

$$\textcircled{27} \text{ (a) } |2x-4| = 3|x-1|$$

$$\Leftrightarrow |2x-4| = |3x-3| \Leftrightarrow 2x-4 = 3x-3 \vee 2x-4 = 3-3x$$

$$\Leftrightarrow x = -1 \vee 5x = 7 \Leftrightarrow \boxed{x = -1} \vee \boxed{x = \frac{7}{5}}$$

$$\text{(B) } |3x-5| > 1 \Leftrightarrow 3x-5 > 1 \vee 3x-5 < -1$$

$$\Leftrightarrow 3x > 6 \vee 3x < 4 \Leftrightarrow x > 2 \vee x < \frac{4}{3}$$

$$\text{(γ) } -1 < \frac{4}{3} \text{ άρα } 1 \leq x < 2. \quad \frac{7}{5} > 2 \text{ άρα } 2 \leq x < \frac{7}{5} \text{ και } \text{(B)}$$

$$\textcircled{28} \text{ (a) } \left. \begin{array}{l} 2 \leq x \leq 3 \\ 1 \leq y \leq 2 \end{array} \right\} \text{(+) } 3 \leq x+y \leq 5$$

$$\text{(B) } \left. \begin{array}{l} 2 \cdot 2 \leq 2x \leq 2 \cdot 3 \Leftrightarrow 4 \leq 2x \leq 6 \\ -3 \geq -3y \geq -6 \Leftrightarrow -6 \leq -3y \leq -3 \end{array} \right\} \text{(+) } -2 \leq 2x-3y \leq 3$$

$$\text{(γ) } \left. \begin{array}{l} 2 \leq x \leq 3 \\ 1 \geq \frac{1}{y} \geq \frac{1}{2} \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} 2 \leq x \leq 3 \\ \frac{1}{2} \leq \frac{1}{y} \leq 1 \end{array} \right\} \text{(*) } 1 \leq \frac{x}{y} \leq 3$$

$$\textcircled{29} \quad (\lambda^2 - 9) \cdot x = \lambda^2 - 3\lambda$$

$$\text{(a) } (\lambda-3)(\lambda+3) \cdot x = (\lambda-3) \cdot \lambda$$

$$\bullet \text{ Για } \lambda = 3 \quad 0x = 0 \text{ ταυτότητα}$$

$$\bullet \text{ Για } \lambda = -3 \quad 0x = (-3-3)(-3) \Leftrightarrow 0x = +18 \text{ αδύνατο}$$

$$\bullet \text{ Για } \lambda = 0 \quad -3(0+3) \cdot x = 0 \Leftrightarrow -9x = 0 \Leftrightarrow \boxed{x=0}$$

(B) Πρέπει $\lambda \neq 3$ και $\lambda \neq -3$

(13)

$$x = \frac{\lambda(\lambda/3)}{(\lambda/3)(\lambda+3)} \Leftrightarrow x = \frac{\lambda}{\lambda+3}$$

(δ) $x=4 \Leftrightarrow \frac{\lambda}{\lambda+3} = \frac{4}{1} \Leftrightarrow \lambda = 4\lambda + 12$

$\Leftrightarrow 3\lambda = -12 \Leftrightarrow \boxed{\lambda = -4}$

30 (a) $a_1 = 1, w = 1$

$$\begin{aligned} S_v &= \frac{v}{2} [2 \cdot a_1 + (v-1) \cdot w] = \frac{v}{2} [2 \cdot 1 + (v-1) \cdot 1] \\ &= \frac{v}{2} (2 + v - 1) = \frac{v(v+1)}{2} \end{aligned}$$

(B) $\frac{v(v+1)}{2} = 45 \Leftrightarrow v^2 + v = 90 \Leftrightarrow v^2 + v - 90 = 0$

$\Leftrightarrow v = 9 \vee v = -10$ ~~απόρρ.~~ $\boxed{v = 9}$

31 (a) $\left| \frac{a}{B} \right| + \left| \frac{B}{a} \right| \geq 2$

$\Leftrightarrow \frac{|a|}{|B|} + \frac{|B|}{|a|} \geq 2 \stackrel{|a| \cdot |B| > 0}{\Leftrightarrow} |a|^2 + |B|^2 - 2|a \cdot B| \geq 0$

$\Leftrightarrow (|a| - |B|)^2 \geq 0$ $\text{το } x \geq 0$

(B) Πρέπει $|a| = |B| \Leftrightarrow a = B \vee a = -B$

$$(39) (a) A_f = A_1 \cup A_2 = (-\infty, 3] \cup (3, 10)$$

$$(B) f(-1) = 2(-1) - 5 = -2 - 5 = -7$$

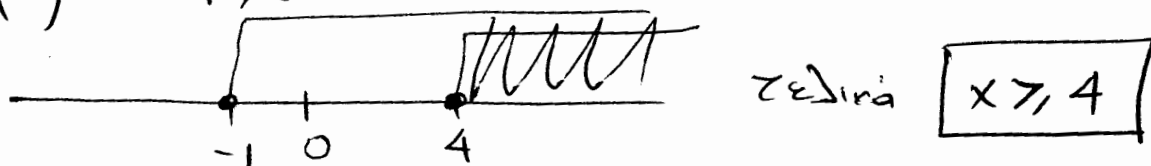
$$f(3) = 2 \cdot 3 - 5 = 6 - 5 = 1$$

$$f(5) = 5^2 = 25$$

$$(γ) f(x) = 25 \quad \text{το} \quad f(5) = 25$$

οπότε $x=5$ άκριβ, $2x-5=25 \Rightarrow x=15$ άπορ $x \leq 3$

$$(33) (a) x-4 \geq 0 \text{ και } x+1 \geq 0 \Leftrightarrow x \geq 4 \text{ και } x \geq -1$$



$$(B) A = (\sqrt{x-4} + \sqrt{x+1})(\sqrt{x-4} - \sqrt{x+1})$$

$$= (\sqrt{x-4})^2 - (\sqrt{x+1})^2 = \cancel{x-4} - \cancel{x-1} = -5$$

$$(34) (a) 3 < \sqrt[3]{30} < 4$$

$$27 < 30 < 64 \Leftrightarrow \sqrt[3]{27} < \sqrt[3]{30} < \sqrt[3]{64}$$

$$\Leftrightarrow 3 < \sqrt[3]{30} < 4$$

$$(B) a = \sqrt[3]{30}, \quad b = 6 - \sqrt[3]{30}$$

$$a-b = \sqrt[3]{30} - 6 + \sqrt[3]{30} = 2\sqrt[3]{30} - 6$$

$$= 2(\sqrt[3]{30} - 3) > 0 \quad \text{οπότε} \quad 3 < \sqrt[3]{30} \quad \text{οπότε} \quad a-b > 0$$

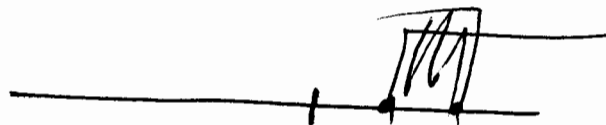
$$\Leftrightarrow a > b \Leftrightarrow \sqrt[3]{30} > 6 - \sqrt[3]{30}$$

$$35 \quad A = \sqrt{x-4} + \sqrt{6-x}$$

(a) Πρέπει $x-4 \geq 0$ και $6-x \geq 0$

$$\Leftrightarrow x \geq 4 \text{ και } x \leq 6$$

Τελικά $4 \leq x \leq 6$ ή $x \in [4, 6]$



$$(b) \quad A = \sqrt{5-4} + \sqrt{6-5} = \sqrt{1} + \sqrt{1} = 1+1 = 2$$

$$A^2 + A - 6 = 2^2 + 2 - 6 = 4 + 2 - 6 = 0$$

$$36 \quad A = \sqrt{x^2+4} - \sqrt{x-4}$$

(a) Πρέπει $x^2+4 \geq 0$ ισχύει $x-4 \geq 0 \Leftrightarrow x > 4$
 $x \in (4, +\infty)$

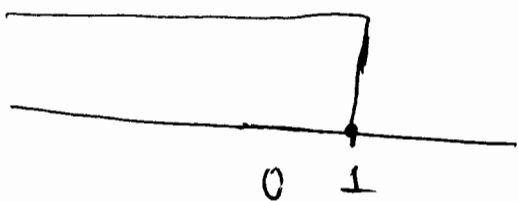
$$(b) \quad A = \sqrt{16+4} - \sqrt{4-4} = \sqrt{20} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$$

$$A^2 - A = 20 - 2\sqrt{5} = 2(10 - \sqrt{5})$$

$$37 \quad A = \sqrt{1-x} - \sqrt[4]{x^4}$$

Πρέπει $1-x \geq 0 \Leftrightarrow x \leq 1$
 και $x^4 \geq 0$ ισχύει πάντα

Τελικά $x \leq 1$



$$(b) \quad A = \sqrt{1+3} - \sqrt[4]{3^4} = 2 - 3 = -1$$

$$A^3 + A^2 + A + 1 = (-1)^3 + (-1)^2 - 1 + 1 = -1 + 1 - 1 + 1 = 0$$

$$(38) (a) B = \sqrt[5]{(x-2)^5} \quad \text{По условию } x-2 \geq 0 \Leftrightarrow x \geq 2$$
$$x \in [2, +\infty)$$

$$(b) B = \sqrt[5]{(4-2)^5} = 2$$

$$B^2 + 6B = 2^2 + 6 \cdot 2 = 4 + 12 = 16 = 2^4 = B^4$$

$$(39) A = (\sqrt{2})^6 = [(\sqrt{2})^2]^3 = 2^3 = 8$$

$$(a) B = \left(\sqrt[3]{2}\right)^6 = \left[\left(\sqrt[3]{2}\right)^3\right]^2 = 2^2 = 4$$

$$A - B = 8 - 4 = 4$$

$$(b) (\sqrt{2})^6 = 8 \quad \text{или} \quad \left(\sqrt[3]{2}\right)^6 = 4$$

$$\text{доп } A - B > 0 \Leftrightarrow \sqrt{2} > \sqrt[3]{2}$$

$$\text{доп } 1 < \sqrt[3]{2} < \sqrt{2}$$

$$(40) (a) |x+1| < 2 \Leftrightarrow -2 < x+1 < 2$$

$$\Leftrightarrow -3 < x < 1 \quad \text{доп } x \in (-3, 1)$$

$$(b) x > -3 \Leftrightarrow x+3 > 0 \quad \text{или} \quad x < 1 \Leftrightarrow x-1 < 0$$

$$\text{доп } K = \frac{x+3 + 1-x}{4} = \frac{4}{4} = 1$$

41 $A = |x-1| + |y-3|$

(a) $x > 1 \Rightarrow x-1 > 0$ or $y < 3 \Rightarrow y-3 < 0$

$\therefore A = x-1 + 3-y \Rightarrow A = x-y+2$

(B) $\left. \begin{array}{l} 1 < x < 4 \\ -3 < -y < -2 \end{array} \right\} \begin{array}{l} (+) -2 < x-y < 2 \Rightarrow 0 < x-y+2 < 4 \\ \Rightarrow 0 < A < 4 \end{array}$

42 (a) $x^2 - 5x + 6 = (x-3)(x-2)$

(B) (i) $D_f = \mathbb{R} - \{2, 3\}$

(ii) $f(x) = \frac{x-2}{(x-3)(x-2)} = \frac{1}{x-3}$

43 $\left[\begin{array}{l} 5A \quad 9M \\ K \cup \Pi = 16 \end{array} \right] \begin{array}{l} (a) (i) P(A) \\ (ii) P(K \cup \Pi) \end{array}$

(B) (i) $P(A) = \frac{N(A)}{N(\Omega)} = \frac{5}{30}$, $N(A) = 5$, $N(\Omega) = 5 + 9 + 16 = 30$

$\therefore P(A') = 1 - \frac{5}{30} = 1 - \frac{1}{6} = \frac{5}{6}$

(ii) $P(K \cup \Pi) = P(K) + P(\Pi)$ $K \cap \Pi = \emptyset$

$P(K) + P(\Pi) = \frac{30-14}{30} = \frac{16}{30}$

$$(44) \quad A = \frac{1+x}{x-1} \quad \text{vai} \quad B = \frac{2}{x^2-x}$$

(a) $\text{Nepeni } x \neq 1 \quad \text{vai} \quad x(x-1) \neq 0 \Rightarrow x \neq 0 \quad \text{vai} \quad x \neq 1$

$$(B) \quad \frac{1+x}{\cancel{x-1}} = \frac{2}{x(\cancel{x-1})} \quad (\Rightarrow) \quad x(1+x) = 2$$

$$\Rightarrow x^2 + x - 2 = 0 \quad x = -2, \quad x = 1 \text{ dropp}$$

$$\boxed{x = -2}$$

$$(45) \quad (a) \quad -2x^2 + 10x - 12 = 0 \quad (\Rightarrow) \quad -2(x^2 - 5x + 6) = 0$$

$$\Rightarrow -2(x-3)(x-2) = 0 \quad \Rightarrow x=3 \quad \text{vai} \quad x=2$$

$$(b) \quad \frac{-2(x-3)(\cancel{x-2})}{\cancel{x-2}} = 0 \quad (\Rightarrow) \quad \boxed{x=3}$$

$$(46) \quad A = |3x-6| + 2 = 3|x-2| + 2$$

(a) (i) $A \vee x \geq 2$ $\text{va} \quad A = 3(x-2) + 2 = 3x - 6 + 2 = 3x - 4$

(ii) $A \vee x < 2$ $\text{va} \quad A = 3(2-x) + 2 = 6 + 2 - 3x = 8 - 3x$

$$(B) \quad \frac{9x^2 - 16}{|3x-6| + 2} = \frac{(3\cancel{x-4})(3x+4)}{\cancel{3x-4}} = 3x+4$$

$$(47) a_2 = 0, a_4 = 4$$

(19)

$$(a) a_2 = a_1 + w = 0 \Leftrightarrow w = -a_1$$

$$a_4 = a_1 + 3w = 4 \Leftrightarrow -w + 3w = 4 \Leftrightarrow 2w = 4 \Leftrightarrow w = 2$$

$$\Leftrightarrow a_1 = -2$$

$$(b) a_n = a_1 + (n-1) \cdot w = -2 + (n-1) \cdot 2 = -2 + 2n - 2$$

$$\Leftrightarrow a_n = 2n - 4$$

$$\exists \in \mathbb{N} \quad a_n = 98 \Leftrightarrow 2n - 4 = 98 \Leftrightarrow 2n = 102 \Leftrightarrow n = 51$$

$$(48) (a) f(1) = 6 \Leftrightarrow a \cdot 1 + B = 6 \Leftrightarrow a + B = 6 \quad (1)$$

$$f(-1) = 4 \Leftrightarrow -a + B = 4 \quad (2)$$

$$(1) + (2) \quad 2B = 10 \Leftrightarrow \boxed{B = 5} \quad \boxed{a = 1}$$

$$f(x) = x + 5$$

$$(b) \cdot \underline{A \text{ για } x'x} : \text{ θέτουμε } f(x) = 0 \Leftrightarrow x = -5, A(-5, 0)$$

$$\cdot \underline{A \text{ για } y'y} : \text{ θέτουμε } x = 0 \Leftrightarrow f(x) = 5, B(0, 5)$$

$$(49) (a) \text{ Πρέπει } (2x+1)^2 = x(5x+4)$$

$$\Leftrightarrow 4x^2 + \cancel{4x} + 1 = 5x^2 + \cancel{4x}$$

$$\Leftrightarrow x^2 = 1 \Leftrightarrow x = 1 \text{ ή } x = -1$$

$$(B) (i) \Gamma \alpha \quad x=1 \quad 1, 3, 9$$

(20)

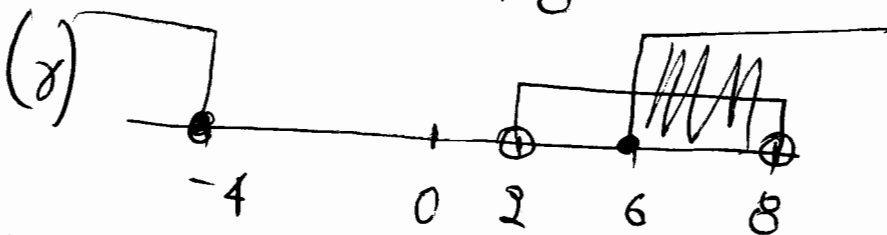
$$\lambda = \frac{3}{1} = 3$$

$$(ii) \Gamma \alpha \quad x=-1, -1, -1, -1$$

$$\lambda = \frac{-1}{-1} = 1$$

$$(50) (a) |x-1| \geq 5 \Leftrightarrow x-1 \geq 5 \vee x-1 \leq -5$$
$$\Leftrightarrow x \geq 6 \vee x \leq -4$$

$$(b) d(x, 5) < 3 \Leftrightarrow |x-5| < 3 \Leftrightarrow -3 < x-5 < 3$$
$$\Leftrightarrow 2 < x < 8$$



$$x \in [6, 8)$$

$$(51) (a) f(-1) = 2(-1) + 4 = 2$$
$$f(3) = 3 - 1 = 2$$

$$\alpha \rho \alpha \quad f(-1) = f(3)$$

$$(B) f(1) = 1 - 1 = 0 \quad \alpha \rho \alpha \quad \gamma \alpha \quad x=1 \quad \Delta \epsilon \kappa \nu \iota \quad \alpha \phi \omega \quad x \geq 0$$
$$2x+4=0 \Leftrightarrow 2x=-4 \Leftrightarrow x=-2 \quad \Delta \epsilon \kappa \nu \iota \quad \alpha \phi \omega \quad x < 0$$

$$(52) (a) (x+1)^2 = \frac{x+2+3x+2}{2} \Leftrightarrow 2(x^2+2x+1) = 4x+4$$

$$\Leftrightarrow x^2 + 2x + 1 = 2x + 2 \Leftrightarrow x^2 = 1 \Leftrightarrow x=1 \vee x=-1$$

$$(B) (i) x=1, 3, 4, 5$$

$$w = 4 - 3 \Rightarrow \boxed{w=1}$$

$$(ii) x=-1, 1, 0, -1$$

$$w = 0 - 1 \Rightarrow \boxed{w=-1}$$

(21)

53

$$(\lambda^2 - 1)x = (\lambda + 1)(\lambda + 2)$$

$$(a) (\lambda - 1)(\lambda + 1) \cdot x = (\lambda + 1)(\lambda + 2)$$

$$\text{Για } \lambda = 1 \quad 0x = 2 \cdot 3 \Rightarrow 0x = 6 \text{ αδύνατη}$$

$$\text{Για } \lambda = -1 \quad 0x = 0 \text{ ταυτολογία}$$

$$(B) \text{ Πρέπει } \lambda \neq 1 \text{ και } \lambda \neq -1$$

$$x = \frac{(\lambda + 1)(\lambda + 2)}{(\lambda - 1)(\lambda + 1)} \Rightarrow \boxed{x = \frac{\lambda + 2}{\lambda - 1}}$$

54

$$a_1 = 120, w = 20$$

$$(a) a_v = a_1 + (v-1) \cdot w = 120 + (v-1) \cdot 20$$

$$\Rightarrow a_v = 120 + 20v - 20 \Rightarrow a_v = 20v + 100$$

$$(B) a_{10} = 20 \cdot 10 + 100 = 200 + 100 = 300$$

$$(γ) S_{10} = \frac{10}{2} [2 \cdot 120 + 9 \cdot 20] = 5 \cdot (240 + 180)$$

$$= 5 \cdot (420) = 2100$$

$$(55) \quad (a) \quad |y-3| < 1 \Leftrightarrow -1 < y-3 < 1$$

$$\Leftrightarrow 2 < y < 4$$

$$(B) \quad 1 < x < 3, \quad 2 < y < 4 \quad E = x \cdot y$$

$$1 \cdot 2 < x \cdot y < 3 \cdot 4 \Leftrightarrow 2 < x \cdot y < 12 \Leftrightarrow 2 < E < 12$$

$$(56) \quad a_1 = 19, \quad a_{10} - a_6 = 24$$

$$(a) \quad a_{10} - a_6 = 24 \Leftrightarrow 9r + 9w - 9r - 5w = 24$$

$$\Leftrightarrow 4w = 24 \Leftrightarrow \boxed{w = 6}$$

$$(B) \quad a_{20} = a_1 + 19 \cdot w = 19 + 19 \cdot 6 = 19(1+6) = 19 \cdot 7$$

$$= 133$$

$$(c) \quad S_{20} = \frac{20}{2} [2 \cdot 19 + 19 \cdot 6] = 10 \cdot (19 \cdot 8) = 1520$$

$$(57) \quad (a) \quad 2x^2 - 3x - 2, \quad \Delta = 9 - 4 \cdot 2 \cdot (-2) = 25$$

$$x_{1,2} = \frac{3 \pm 5}{4} \quad \begin{cases} 2 \\ -\frac{1}{2} \end{cases}$$

$$\text{d'où} \quad 2(x-2)(x+\frac{1}{2}) = (x-2)(2x+1)$$

$$(B) \quad D_K = \mathbb{R} - \left\{ -\frac{1}{2}, 2 \right\}$$

$$(c) \quad K = \frac{(x-2)^2}{\cancel{(x-2)}(2x+1)} = \frac{x-2}{2x+1}$$

$$(58) (a) \frac{a+B}{B} = 4 \Leftrightarrow a+B = 4B \Leftrightarrow \boxed{a = 3B} \quad (23)$$

$$\frac{\gamma}{\delta - \gamma} = \frac{1}{4} \Leftrightarrow \delta - \gamma = 4\gamma \Leftrightarrow \boxed{5\gamma = \delta}$$

$$(B) \pi = \frac{a\gamma + B \cdot \gamma}{B \cdot \delta - B \cdot \gamma} = \frac{\gamma(a+B)}{B(\delta - \gamma)} = \frac{\gamma(3B+B)}{B(5\gamma - \gamma)}$$
$$= \frac{\cancel{\gamma} \cdot 4B}{\cancel{B} \cdot 4\gamma} = 1$$

$$(59) (a) |y-3| < 1 \Leftrightarrow -1 < y-3 < 1 \Leftrightarrow 2 < y < 4$$

$$(B) 1 < x < 3, \quad 2 < y < 4$$

$$\left. \begin{array}{l} 2 < 2x < 6 \\ 4 < 2y < 8 \end{array} \right\} \Leftrightarrow 6 < \pi < 14$$

$$(60) (a) |x-5| < 4 \Leftrightarrow -4 < x-5 < 4 \Leftrightarrow 1 < x < 9$$

$$(B) 1 < a < 9 \Leftrightarrow \frac{1}{1} > \frac{1}{a} > \frac{1}{9} \Leftrightarrow \frac{1}{9} < \frac{1}{a} < 1$$

$$(61) (a) \frac{4x+5y}{x-4y} = -2 \Leftrightarrow 4x+5y = -2x+8y$$

$$\Leftrightarrow 6x = 3y \Leftrightarrow \boxed{y = 2x}$$

$$(B) A = \frac{2x^2 + 3 \cdot 4x^2 + x \cdot 2x}{x \cdot 2x} = \frac{2x^2 + 12x^2 + 2x^2}{2x^2} \quad (24)$$

$$= \frac{16x^2}{2x^2} \Rightarrow A = 8$$

$$(62) (a) x^2 - x - 6 \neq 0 \quad x = 3 \quad \vee \quad x = -2$$

$$D_f = \mathbb{R} - \{-2, 3\}$$

$$(B) f(x) = \frac{\cancel{(x+2)}}{\cancel{(x+2)}(x-3)} = \frac{1}{x-3}$$

$$f(2) = \frac{1}{2-3} = -1, \quad f(4) = \frac{1}{4-3} = 1$$

$$\text{d'où } f(2) + f(4) = -1 + 1 = 0$$

$$(63) (a) B = \frac{a+y}{2} \Rightarrow x+4 = \frac{1+x+3}{2}$$

$$\Rightarrow 2x+8 = x+9 \Rightarrow \boxed{x=1}$$

$$(B) A=1, B=5, \Gamma=9$$

$$(i) a_1 = 1 \quad w = 5 - 1 \Rightarrow w = 4$$

$$(ii) a_{20} = a_1 + 19w = 1 + 19 \cdot 4 = 1 + 76 = 77$$

64 (a) $4-x, x, 2$

$$x = \frac{4-x+2}{2} \Leftrightarrow 2x = 6-x \Leftrightarrow 3x = 6 \Leftrightarrow \boxed{x=2}$$

$2, 2, 2$

(B) $x^2 = (4-x) \cdot 2 \Leftrightarrow x^2 = 8 - 2x$

$$\Leftrightarrow x^2 + 2x - 8 = 0 \quad x_1 = -4, x_2 = 2$$

$8, -4, 2, 2, 2, 2$

(r) $\text{перенеси } \boxed{x=2}$

65 $5 < x < 10$

(a) $5 < x \Leftrightarrow x-5 > 0 \quad \text{де } |x-5| = x-5$

$x < 10 \Leftrightarrow x-10 < 0 \quad \text{де } |x-10| = 10-x$

(B) $A = \frac{x-5}{x-5} + \frac{-(x-10)}{x-10} = 1 - 1 = 0$

66 (a) $D_f = \mathbb{R} - \{-1, 1\}$

(B) $f(a) = \frac{1}{8} \Leftrightarrow \frac{1}{a^2-1} = \frac{1}{8}$

$$\Leftrightarrow a^2 - 1 = 8 \Leftrightarrow a^2 = 9 \Leftrightarrow a = 3 \vee a = -3$$

$M(3, \frac{1}{8}), M'(-3, \frac{1}{8})$

$$(67) \quad (a) \quad A = \underset{+}{|x-1|} - \underset{-}{|x-2|} = x-1 - (2-x)$$

$$1 < x \Leftrightarrow x-1 > 0, \quad x < 2 \Leftrightarrow x-2 < 0$$

$$= x-1 + x-2 = 2x-3$$

$$(B) \quad x < 1 \Leftrightarrow x-1 < 0, \quad x-2 < -1 < 0$$

$$A = 1-x - (2-x) = 1-x + x-2 = -1$$

$$\forall x \quad A = -1$$

$$(68) \quad (a) \quad AB + BZ + ZE + E\Delta + \Delta\Gamma + \Gamma A$$

$$= x + 2y + x - \cancel{y} + y + \cancel{y} + y = 2x + 4y$$

$$(B) \quad 10 < 2x < 16 \quad \text{and} \quad 4 < 4y < 8$$

$$14 < 2x + 4y < 24 \Leftrightarrow 14 < \pi < 24$$

$$(69) \quad (a) \quad A+B = \frac{1}{5+\sqrt{5}} + \frac{1}{5-\sqrt{5}}$$

$$= \frac{5-\sqrt{5} + 5+\sqrt{5}}{25-5} = \frac{10}{20} = \frac{1}{2}$$

$$A \cdot B = \frac{1}{(5+\sqrt{5})(5-\sqrt{5})} = \frac{1}{25-20} = \frac{1}{5}$$

$$(B) \quad x^2 - 5x + P = 0 \Leftrightarrow x^2 - \frac{1}{2}x + \frac{1}{20} = 0$$

$$\Leftrightarrow 20x^2 - 10x + 1 = 0$$

$$\textcircled{70} \quad y = 35 + 0,8 \cdot x$$

$$\text{(a)} \quad \text{für } x = 25$$

$$y = 35 + 0,8 \cdot 25 \Rightarrow y = 35 + \frac{8}{10} \cdot \frac{5}{2}$$

$$\Rightarrow y = 35 + 20 \Rightarrow \boxed{y = 55 \text{ km}}$$

$$\text{(B)} \quad y = 75 \Rightarrow 35 + 0,8 \cdot x = 75$$

$$\Rightarrow 0,8 \cdot x = 40 \Rightarrow x = \frac{40}{0,8} \Rightarrow x = 50 \text{ min}$$

$$\textcircled{71} \text{(a)} \quad 2 \cdot 1^2 + \lambda \cdot 1 - 5 = 0 \Rightarrow 2 + \lambda - 5 = 0$$

$$\Rightarrow \boxed{\lambda = 3}$$

$$\text{(B)} \quad 2x^2 + 3x - 5 = 0 \quad \Delta = 9 - 4 \cdot 2 \cdot (-5) = 9 + 40$$

$$x_{1,2} = \frac{-3 \pm 7}{4} \begin{cases} 1 \\ -\frac{5}{2} \end{cases}$$

$$2(x-1)\left(x+\frac{5}{2}\right) = (x-1)(2x+5)$$

$$\textcircled{72} \quad 2x^2 - 5Bx + 2B^2 = 0 \quad B > 0$$

$$\text{(a)} \quad S = \frac{5B}{2}, \quad P = B^2$$

$$x_1 + x_2 = 2B + \frac{B}{2} = \frac{4B + B}{2} = \frac{5B}{2}$$

$$x_1 \cdot x_2 = 2B \cdot \frac{B}{2} = B^2 \quad | \text{ } x_i \in \mathbb{R}$$

$$(B) \quad 2B, B, \frac{B}{2}$$

(28)

$$B^2 = 2B \cdot \frac{B}{2} \Rightarrow B^2 = B^2 \quad \text{10x34}$$

$$(73) \quad x^2 - 2Bx + (B^2 - 4) = 0 \quad (1)$$

$$(a) \quad S = 2B, \quad P = B^2 - 4$$

$$x_1 + x_2 = B - 2 + B + 2 = 2B = S$$

$$x_1 \cdot x_2 = (B - 2)(B + 2) = B^2 - 4 = P \quad \text{10x34}$$

$$(B) \quad B - 2, B, B + 2$$

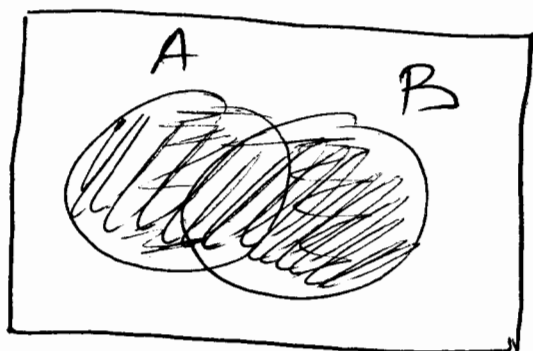
$$B = \frac{B - 2 + B + 2}{2} \Rightarrow B = B \quad \text{10x34}$$

$$(74) \quad P(A) = \frac{3}{4}, \quad P(A-B) = \frac{5}{8} \quad \text{ka}, \quad P(B) = \frac{1}{4}$$

$$(a) \quad P(A-B) = \frac{5}{8} \Rightarrow P(A) - P(A \cap B) = \frac{5}{8}$$

$$\Rightarrow \frac{6}{8} - P(A \cap B) = \frac{5}{8} \Rightarrow P(A \cap B) = \frac{1}{8}$$

(B)(i)



$$A \cup B = \{x \in \Omega / x \in A \vee x \in B\}$$

$$(ii) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{6}{8} + \frac{2}{8} - \frac{1}{8} = \frac{7}{8}$$

(29)

75 $|x-3| \leq 2 \Rightarrow -2 \leq x-3 \leq 2$

(a) $\Rightarrow 1 \leq x \leq 5$

$$|y-6| \leq 4 \Rightarrow -4 \leq y-6 \leq 4$$

$$\Rightarrow 2 \leq y \leq 10$$

(B) $\Pi = 2 \cdot 2x + 2y = 4x + 2y$

$$\left. \begin{array}{l} 4 \leq 4x \leq 20 \\ 4 \leq 2y \leq 20 \end{array} \right\} (+) \quad 8 \leq \Pi \leq 40$$

76 $2x^2 + 5x - 1$

(a) $\Delta = 25 - 4 \cdot 2(-1) = 25 + 8 = 33 > 0$

(B) $S = x_1 + x_2 = -\frac{5}{2}, \quad P = x_1 \cdot x_2 = \frac{-1}{2}$

$$\frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 \cdot x_2} = \frac{-\frac{5}{2}}{-\frac{1}{2}} = 5$$

(\gamma) $S' = \frac{1}{x_1} + \frac{1}{x_2} = 5, \quad P' = \frac{1}{x_1 \cdot x_2} = -2$

da $x^2 - S'x + P' = 0 \Rightarrow x^2 - 5x - 2 = 0$

$$(77) (A) D_k = \mathbb{R} - \{-2, 3\}$$

$$(B) k = \frac{\sqrt{(x+2)^2}}{x+2} - \frac{\sqrt{(x-3)^2}}{x-3} = \frac{|x+2|}{x+2} - \frac{|x-3|}{x-3}$$

$$-2 < x < 3 \text{ da } x+2 > 0 \text{ kai } x-3 < 0$$

$$= \frac{\cancel{x+2}}{\cancel{x+2}} - \frac{-(\cancel{x-3})}{\cancel{x-3}} = 1 + 1 = 2$$

$$(78) -x^2 + 5x - 6 < 0 \text{ kai } x^2 - 16 \leq 0$$

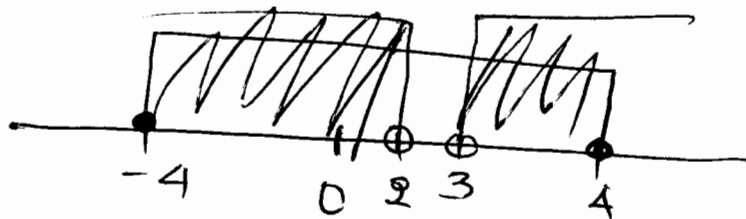
$$\Leftrightarrow x^2 - 5x + 6 > 0$$

$$|x|^2 \leq 16$$

$$\Leftrightarrow |x| \leq 4 \Leftrightarrow -4 \leq x \leq 4$$

| | | | | |
|----------------|-----------|-------------|-------------|-----------|
| x | $-\infty$ | 2 | 3 | $+\infty$ |
| $x^2 - 5x + 6$ | + | \emptyset | \emptyset | + |

$$x \in (-\infty, 2) \cup (3, +\infty)$$



$$\text{da } x \in [-4, 2) \cup (3, 4]$$

$$(79) (a) d(x, -2) < 1 \Leftrightarrow |x+2| < 1 \Leftrightarrow -1 < x+2 < 1$$

$$\Leftrightarrow -3 < x < -1$$

$$(B) x^2 + 4x + 3 < 0 \quad \Delta = 16 - 12 = 4 \quad X_{1,2} = \frac{-4 \pm 2}{2} \begin{matrix} -1 \\ -3 \end{matrix}$$

$$\Leftrightarrow (x+1)(x+3) < 0$$

$$x < -1 \Leftrightarrow x+1 < 0$$

$$x > -3 \Leftrightarrow x+3 > 0$$

10 x 0 1.

| | | | | |
|----------------|-----------|-------------|-------------|-----------|
| x | $-\infty$ | -3 | -1 | $+\infty$ |
| $x^2 + 4x + 3$ | + | \emptyset | \emptyset | + |

$$\textcircled{80} \quad -x^2 + (\sqrt{3}-1)x + \sqrt{3} \quad a=-1, B=\sqrt{3}-1, \gamma=\sqrt{3} \quad (31)$$

$$(a) \quad \Delta = (\sqrt{3}-1)^2 - 4 \cdot (-1) \cdot \sqrt{3} = 3 - 2\sqrt{3} + 1 + 4\sqrt{3} \\ = 4 + 2\sqrt{3} = (\sqrt{3}+1)^2$$

$$(B) \quad x_1 = \frac{1 - \sqrt{3} + (\sqrt{3}+1)}{-2} = \frac{1 - \sqrt{3} + \sqrt{3} + 1}{-2} = -1$$

$$x_2 = \frac{1 - \sqrt{3} - \sqrt{3} - 1}{-2} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$$

$$\text{d'ed} \quad - (x+1)(x-\sqrt{3})$$

$$\textcircled{81} \quad (a) \quad 3x^2 - 2x - 1 \quad \Delta = 4 - 4 \cdot 3 \cdot (-1) = 4 + 12 = 16$$

$$x_{1,2} = \frac{2 \pm 4}{6} \begin{cases} 1 \\ -\frac{1}{3} \end{cases}$$

$$3(x-1)(x+\frac{1}{3}) = (x-1)(3x+1)$$

$$(B) \quad \text{Peenel} \quad x \neq 1 \quad \text{ka} \quad x \neq -\frac{1}{3}$$

$$x \in \mathbb{R} - \left\{ 1, -\frac{1}{3} \right\}$$

$$A(x) = \frac{x-1}{(x-1)(3x+1)} = \frac{1}{3x+1}$$

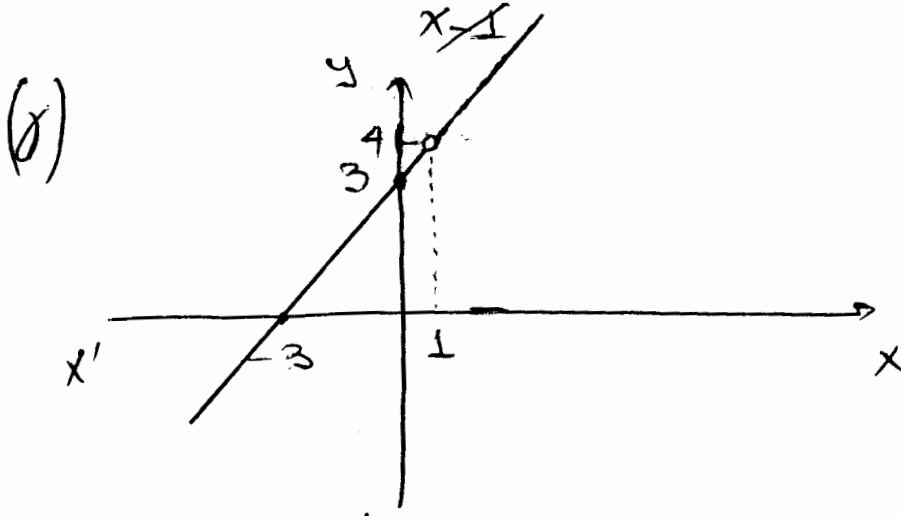
$$(x) \quad \frac{1}{|3x+1|} = 1 \quad (\Leftrightarrow) \quad |3x+1| = 1 \quad (\Leftrightarrow) \quad 3x+1 = 1$$

$$3x+1 = -1$$

$$\Leftrightarrow \quad x=0 \quad \vee \quad x = -\frac{2}{3}$$

82 (a) $x^2 + 2x - 3$ $x = -3, x = 1$

(B) $f(x) = \frac{(x+3)(x-1)}{x-1} = x+3$ $\text{def } D_f = \mathbb{R} - \{1\}$



83

$$P(A) = \frac{N(A)}{N(\Omega)} = \frac{3}{9} = \frac{1}{3}$$

$$P(B) = \frac{N(B)}{N(\Omega)} = \frac{1}{9}$$

$$P(A \cup B) = \frac{1}{3} + \frac{3}{9} - \frac{1}{9} = \frac{3}{9} + \frac{3}{9} - \frac{1}{9} = \frac{5}{9}$$

84 (a) $x^2 - 10x + 21 < 0$ $x \in (3, 7)$

| | | | | |
|------------------|-----------|-----|-----|-----------|
| x | $-\infty$ | 3 | 7 | $+\infty$ |
| $x^2 - 10x + 21$ | $+$ | 0 | 0 | $+$ |

(B) (i) $A = |x-3| + |x^2 - 10x + 21|$

$$= x-3 - x^2 + 10x - 21 = -x^2 + 11x - 24$$

(ii) $-x^2 + 11x - 24 = 6 \Leftrightarrow x^2 - 11x + 30 = 0$

$$x = 5 \vee x = 6 \quad \Delta \epsilon \kappa \tau \epsilon \varsigma$$

85 $T = 15 + 25 \cdot x$, $0 \leq x \leq 200$

(a) $x = 30$, $T = 15 + 25 \cdot 30 \Rightarrow T = 15 + 750 = 765^\circ$

(b) $T = 290 \Leftrightarrow 15 + 25 \cdot x = 290 \Leftrightarrow 25 \cdot x = 275$

$\Leftrightarrow x = 11 \text{ km}$

(c) $T > 440^\circ \Leftrightarrow 15 + 25 \cdot x > 440^\circ \Leftrightarrow 25 \cdot x > 425$

$\Leftrightarrow \boxed{x > 17 \text{ km}}$

86

(a) $3x^2 - 4x + 1 \leq 0$ $\Delta = 16 - 4 \cdot 3 \cdot 1 = 16 - 12 = 4$

$x_{1,2} = \frac{4 \pm 2}{6} \begin{cases} 1 \\ \frac{1}{3} \end{cases}$

$x \in \left[\frac{1}{3}, 1 \right]$

| | | | | | |
|-----------------|-----------|---------------|-----|-----------|-----|
| x | $-\infty$ | $\frac{1}{3}$ | 1 | $+\infty$ | |
| $3x^2 - 4x + 1$ | $+$ | 0 | $-$ | 0 | $+$ |

(b) $\frac{1}{3} \leq a \leq 1$ ca $\frac{1}{3} \leq b \leq 1$

da $1 \leq 3a \leq 3$ ca $2 \leq 6b \leq 6$

$3 \leq 3a + 6b \leq 9 \Leftrightarrow \frac{3}{9} \leq \frac{3a + 6b}{9} \leq 1$

$\Leftrightarrow \frac{1}{3} \leq \frac{3a + 6b}{9} \leq 1$

da $\frac{3a + 6b}{9} \in \left[\frac{1}{3}, 1 \right]$

87 (a) $aB(a+B) = -30 \Rightarrow a \cdot B = \frac{-30}{2}$
 $\Rightarrow a \cdot B = -15$

(B) $x^2 - 2x - 15 = 0 \quad a = 5, B = -3$

88 (a) $A = [(\sqrt{2})^2]^3 = 2^3 = 8, B = [(\sqrt[3]{3})^3]^2 = 3^2 = 9$
 $\Gamma = (\sqrt[6]{6})^6 = 6$

$A+B+\Gamma = 8+9+6 = 23$

(B) $(\sqrt[3]{3})^6 = 9 \text{ kai } (\sqrt[6]{6})^6 = 6$

$\therefore \sqrt[3]{3} > \sqrt[6]{6}$

89 (a) $a_1 - a_2 = 10 \Rightarrow a_1 + 3\omega - a_1 - \omega = 10$

$\Rightarrow 2\omega = 10 \Rightarrow \boxed{\omega = 5}$

(B) $S_3 = 33 \Rightarrow \frac{3}{2} [2a_1 + 2 \cdot 5] = 33$

$\Rightarrow 3 [2a_1 + 10] = 66 \Rightarrow 2a_1 + 10 = 22$

$\Rightarrow 2a_1 = 12 \Rightarrow \boxed{a_1 = 6} \quad 6 \xrightarrow{+5} 11 \xrightarrow{+5} 16$

90 (a) $f(-5) = 8+5 = 13, f(4) = 8+5 = 13$

(B) • Av $x < 0 \quad 8-x = 9 \Rightarrow x = -1 \Delta \epsilon \kappa \mu$

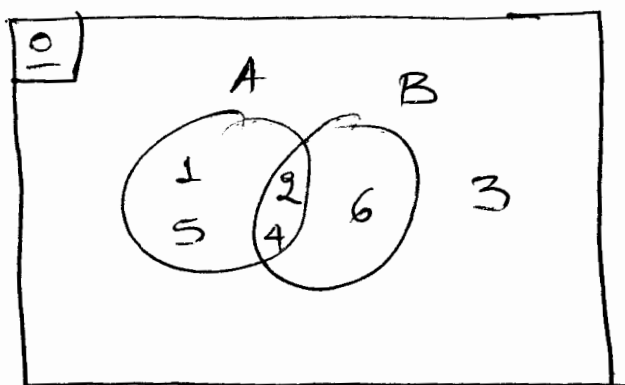
• Av $x \geq 0 \quad 2x+5 = 9 \Rightarrow 2x = 4 \Rightarrow x = 2 \Delta \epsilon \kappa \mu$

91 (a) $|x+4| \geq 3 \Leftrightarrow x+4 \geq 3 \vee x+4 \leq -3$
 $\Leftrightarrow x \geq -1 \vee x \leq -7$

(B) $A = \{ |a+4| - 3 \mid a+1 \geq 0 \}$ $\Leftrightarrow a+4 \geq 3 > 0$

$$A = \{ |a+4-3| = |a+1| = a+1 \}$$

92 (a)



$$A \cup B = \{1, 2, 4, 5, 6\}$$

$$A \cap B = \{2, 4\}$$

$$A' = \{3, 6\}$$

$$B' = \{1, 3, 5\}$$

(B) (i) $P(A) = \frac{2}{6} = \frac{1}{3}$ (ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

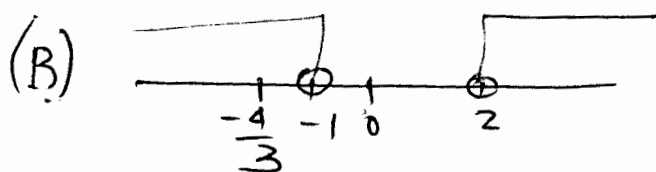
(ii) $P(A \cap B) = \frac{2}{6} = \frac{1}{3}$ $= \frac{4}{6} + \frac{3}{6} - \frac{2}{6} = \frac{5}{6}$

93 (a) $x^2 - (\lambda-1)x + 6 = 0 \Leftrightarrow -\lambda+1 = -6-1$
 $\Leftrightarrow \lambda = 8$

(B) $x^2 - x + 6 = 0 \quad \Delta = 1 - 24 < 0$ Δ δ ν ν ν

94 (a) $x^2 - x - 2 = 0 \quad x = 2, x = -1$

| | | | | |
|---------------|-----------|------|-----|-----------|
| x | $-\infty$ | -1 | 2 | $+\infty$ |
| $x^2 - x - 2$ | + | - | + | + |



$\epsilon_{\text{N21}} \Lambda \Upsilon \Sigma \text{H}$

95 (a) $a_3 = a_1 + 2w = 9 \Leftrightarrow 1 + 2w = 9 \Leftrightarrow 2w = 8 \Leftrightarrow w = 4$

(B) $a_v > 30 \Leftrightarrow a_1 + (v-1) \cdot w > 30$

$\Leftrightarrow 1 + (v-1) \cdot 4 > 30 \Leftrightarrow 4(v-1) > 29$

$\Leftrightarrow 4v - 4 > 29 \Leftrightarrow 4v > 33 \Leftrightarrow v > \frac{33}{4}$

d/c $v = 9$

96 $P(A) = 50\%$ $P(B) = 40\%$ $P(A \cap B) = 10\%$

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 50\% + 40\% - 10\% = 80\%$

(B) $P(A' \cap B') = 1 - P(A \cup B) = 20\%$

97 (a) $f(0) = 5 \Leftrightarrow a \cdot 0 + b = 5 \Leftrightarrow b = 5$

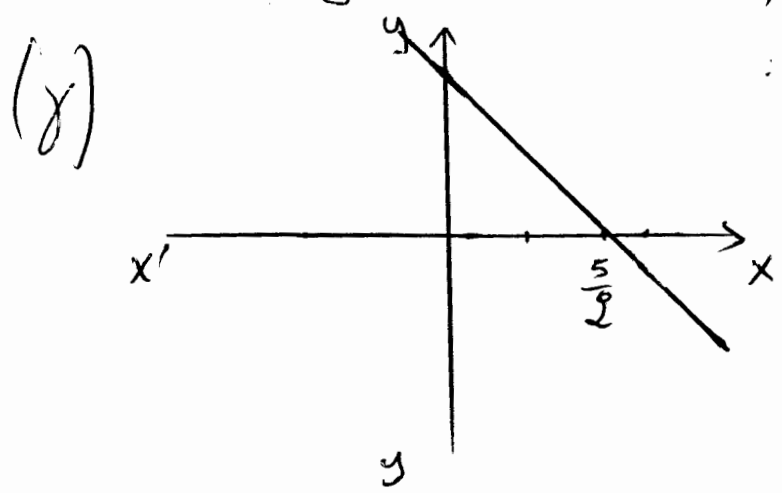
$f(1) = 3 \Leftrightarrow a + 5 = 3 \Leftrightarrow a = -2$

(B) $f(x) = -2x + 5$

• A ζova x'x δετουμε $f(x) = 0 \Leftrightarrow 2x = 5 \Leftrightarrow x = \frac{5}{2}$

$A(\frac{5}{2}, 0)$

• A ζova y'y δετουμε $x = 0, f(x) = 5, B(0, 5)$



98 (a) $f = \mathbb{R} - \{4\}$

$$f(x) = \frac{x(x^2 - 16)}{x - 4} = \frac{x(x-4)(x+4)}{x-4} = x^2 + 4x$$

(b) $x^2 + 4x - 32 = 0 \quad x = -8, x = 4$ *drop*

$\therefore x = -8$

99 $x^2 + 2x + \lambda - 2 = 0 \quad \alpha = 1, \beta = 2, \gamma = \lambda - 2$

(a) $\Delta = 4 - 4 \cdot 1 (\lambda - 2) \geq 0 \Leftrightarrow 4 - 4\lambda + 8 \geq 0$

$\Leftrightarrow -4\lambda \geq -12 \Leftrightarrow \lambda \leq 3$

(b) $x_1 + x_2 = -2, \quad x_1 \cdot x_2 = \lambda - 2$

$\lambda - 2 - 2(-2) = 1 \Leftrightarrow \lambda - 2 + 4 = 1$

$\Leftrightarrow \lambda = -1$

100 (a) $f\left(\frac{1}{2}\right) = \frac{1}{2} + 2 = \frac{1}{2} + \frac{4}{2} = \frac{5}{2}$

$f(1) = 1 + 1 = 2, \quad f(2) = 2 + \frac{1}{2} = \frac{5}{2}$

$\therefore A = \frac{5}{2} + 2 - \frac{5}{2} = 2$

(b) $x + \frac{1}{x} = \frac{5}{2} \Leftrightarrow 2x^2 + 2 = 5x \Leftrightarrow 2x^2 - 5x + 2 = 0$

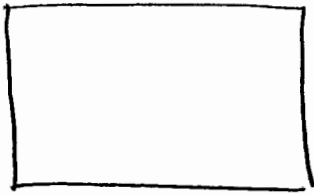
$\Leftrightarrow x^2 - \frac{5}{2}x + 1 = 0, \quad x = \frac{1}{2} \vee x = 2$

101 (a) $4 \leq x \leq 7$ kai $2 \leq y \leq 3$

$$\pi = 2x + 2y$$

$$8 \leq 2x \leq 14 \text{ kai } 4 \leq 2y \leq 6$$

$$12 \leq \pi \leq 20$$

(B)  $\pi' = 2(x-1) + 2 \cdot 3y$
 $= 2x - 2 + 6y$

$$2 \leq y \leq 3 \Rightarrow 12 \leq 6y \leq 18$$

$$20 \leq 2x + 6y \leq 38 \Rightarrow 18 \leq \pi' \leq 36$$

102 (a) $A = x^3 - x^2 + 3x - 3 = x^2(x-1) + 3(x-1)$
 $= (x-1)(x^2+3)$

(B) $f(x) = g(x) \Rightarrow \frac{3}{x} = x^2 - x + 3$

$$\Rightarrow 3 = x^3 - x^2 + 3x \Rightarrow x^3 - x^2 + 3x - 3 = 0$$

$$\Rightarrow (x-1)(x^2+3) = 0 \Rightarrow \boxed{x=1}$$

$$\text{apax } y = 3 \quad A(1, 3)$$

103 (a) $x^2 + 4x + 5 > 0 \quad \Delta = 16 - 20 < 0$

(B) $B = x^2 + 4x + 5 - |(x+2)^2|$
 $= x^2 + 4x + 5 - (x^2 + 4x + 4)$
 $= x^2 + 4x + 5 - x^2 - 4x - 4 = 1$

104

$$(a) f(x) = g(x) \Leftrightarrow x^3 = x \Leftrightarrow x^3 - x = 0$$

$$\Leftrightarrow x(x^2 - 1) = 0 \Leftrightarrow x = 0 \vee x = 1 \vee x = -1$$

$$A(0,0), B(1,1), \Gamma(-1,-1)$$

(B) Έχουν αντίθετες τετραγωνικές και τετραγώνισμα

105

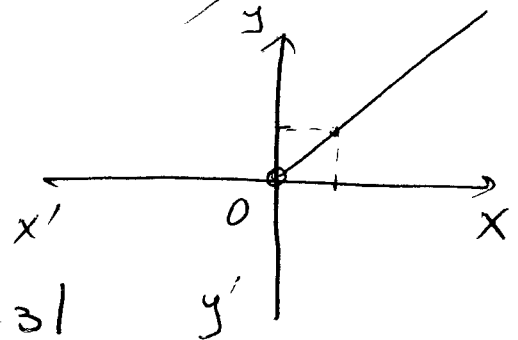
$$(a) \text{ Πρέπει } 2|x| - 6 \neq 0 \Leftrightarrow 2|x| \neq 6 \Leftrightarrow |x| \neq 3$$

$$\Leftrightarrow x \neq 3 \text{ και } x \neq -3$$

$$D_f = \mathbb{R} - \{-3, 3\}$$

$$(B) f(x) = \frac{2|x|^2 - 6|x|}{2|x| - 6} = \frac{|x|(\cancel{2|x| - 6})}{2|x| - 6} = |x|$$

$$(g) f(x) = \begin{cases} x, & x > 0 \\ -x, & x \leq 0 \end{cases}$$



106

$$(a) A+B = 2|x-2| + |x-3| \quad y'$$

$$x \geq 2 \Leftrightarrow x-2 \geq 0 \text{ και } x < 3 \Leftrightarrow x-3 < 0$$

$$\text{οπότε } A+B = 2(x-2) + 3-x = 2x-4+3-x = x-1$$

$$(B) \text{ Θέλουμε } x-1 = 2 \Leftrightarrow x = 3 \text{ όμως } 3 \notin [2, 3)$$

107 (a) $D_f = [-2, 6]$

(B)

| | | | | | | |
|---|----|----|----|---|---|----|
| x | -2 | -1 | 0 | 1 | 2 | 6 |
| y | 3 | 0 | -1 | 0 | 3 | -3 |

(γ) $A \int_{\text{ova}} x'x \quad A(1, 0), B(-1, 0)$

$A \int_{\text{ova}} y'y \quad \Gamma(0, -1)$

(δ) $(-1, 1) \cup (4, 6]$

108 (a) $D_f = [-3, 8]$

(B)

| | | | | | | |
|---|----|----|---|---|----|----|
| x | -3 | -1 | 0 | 3 | 7 | 8 |
| y | 0 | 2 | 3 | 6 | -2 | -4 |

(γ) $A \int_{\text{ova}} x'x \quad A(-3, 0), B(6, 0)$

$A \int_{\text{ova}} y'y \quad \Gamma(0, 3)$

(δ) $(-3, 6)$

109 (a) $f(x) = 3x^2 + 9x - 12 = 3(x^2 + 3x - 4)$
 $= 3(x+4)(x-1)$

| | | | | |
|------------------|-----------|------|-----|-----------|
| x | $-\infty$ | -4 | 1 | $+\infty$ |
| $3x^2 + 9x - 12$ | $+$ | 0 | $-$ | $+$ |

$x \in [-4, 1] \quad \wedge \quad -4 \leq x \leq 1$

$0 \leq 9 \leq 8 \quad (\Rightarrow) \quad 0 \leq \sqrt[3]{9} \leq 2$

dep α to $\sqrt[3]{9}$ Sen enai \int con ms arivawms

110

$$(a) g(1) = -4 \Leftrightarrow \frac{2 \cdot 1^2 - 4 \cdot 1 + r}{1+1} = -4$$

$$\Leftrightarrow -2 + r = -8 \Leftrightarrow \boxed{r = -6}$$

$$(B) D_g = \mathbb{R} - \{-1\}$$

$$(g) g(x) = \frac{2x^2 - 4x - 6}{x+1} = \frac{2(x^2 - 2x - 3)}{x+1} \quad \Delta = 4 + 12 = 16$$

$$x_{1,2} = \frac{2 \pm 4}{2} \begin{matrix} \rightarrow 3 \\ \rightarrow -1 \end{matrix}$$

$$= \frac{2(x-3)(x+1)}{x+1} = 2(x-3)$$

111

$$(a) A = \frac{\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{3}(\sqrt{5}+\sqrt{3})}{5-3} + \frac{\sqrt{5}(\sqrt{5}-\sqrt{3})}{5-3}$$

$$= \frac{\sqrt{15} + 3 + 5 - \sqrt{15}}{2} = 4$$

$$(B) |x+4| = 1 \Leftrightarrow x+4 = 1 \vee x+4 = -1$$

$$\Leftrightarrow x = -3 \vee x = -5$$

112

(a) (i) AUM να έχει αυτοκίνητο ή μηχανάκι

(ii) M-A να έχει μηχανάκι και όχι αυτοκίνητο

(iii) M' να μην έχει μηχανάκι

$$(B) (i) P(M') = 1 - P(M) = 100 - 40\% = 60\%$$

$$(ii) P(A' \cap M') = P[\overline{(A \cup M)}] = 1 - P(A \cup M) = 10\%$$

$$P(A \cup M) = P(A) + P(M) - P(A \cap M) = 70\% + 40\% - 20\% = 90\%$$

113

- (a) (i) 0 πιθανώς συγγράφει στη θ.ο ή 0Σ
- (ii) 0 " " " " στην 0Σ και όχι στην θ.ο
- (iii) 0 πιθανώς δεν συγγράφει στη θ.ο

(b) (i) $P(A) = \frac{20}{180}$, $P(B) = \frac{30}{180}$, $P(A \cap B) = \frac{10}{180}$

$P(A \cup B) = \frac{20 + 30 - 10}{180} = \frac{40}{180}$

$P(A \cap B') = 1 - P(A \cup B) = 1 - \frac{40}{180} = \frac{140}{180}$

(ii) $P(B - A) = P(B) - P(A \cap B) = \frac{30}{180} - \frac{10}{180} = \frac{20}{180}$

114

(a) $(2k)^2 = (k-2)(7k+4) \Rightarrow 4k^2 = 7k^2 + 4k - 14k - 8$

$\Rightarrow 3k^2 - 10k - 8 = 0 \quad \Delta = 100 + 96 = 196$

$k_{1,2} = \frac{10 \pm 14}{6} \begin{cases} 4 \\ -\frac{1}{3} \end{cases}$ απορρ αφοω $k \in \mathbb{Z}$

2, 8, 32 αρα $\lambda = 4$

(b)(i) $a_2 = a_1 \cdot \lambda \Rightarrow a_2 = 4a_1$ $a_3 = 32a_1$

$a_4 = a_1 \cdot \lambda^3 \Rightarrow a_4 = 8a_1$

(ii) $4a_1 + 32a_1 = 36a_1$

$4(a_1 + a_4) = 4a_1 + 4 \cdot 8a_1 = 36a_1$

115

$$(a) \lambda(-2)^2 - (\lambda-1)(-2) - 1 = 0$$

$$\Leftrightarrow 4\lambda + 2\lambda - 2 - 1 = 0 \Leftrightarrow 6\lambda = 3 \Leftrightarrow \lambda = \frac{1}{2}$$

$$(B) \Delta = (\lambda-1)^2 - 4 \cdot \lambda(-1) = \lambda^2 - 2\lambda + 1 + 4\lambda \\ = \lambda^2 + 2\lambda + 1 = (\lambda+1)^2 \geq 0$$

116

$$(a) \Delta = 4\lambda^2 - 4(\lambda+2)(\lambda-1) = 4\lambda^2 - 4(\lambda^2 - \lambda + 2\lambda - 2) \\ = 4\lambda^2 - 4\lambda^2 - 4\lambda - 2$$

$$\text{Пренеси } \Delta > 0 \Leftrightarrow -4\lambda - 2 > 0 \Leftrightarrow 4\lambda < -2 \Leftrightarrow \lambda < -\frac{1}{2}$$

$$(B) S = 2 \Leftrightarrow -\frac{B}{a} = 2 \Leftrightarrow -\frac{2\lambda}{\lambda+2} = 2$$

$$\Leftrightarrow -2\lambda = 2\lambda + 4 \Leftrightarrow 4\lambda = -4 \Leftrightarrow \lambda = -1$$

117

$$(a) 2 \leq a \leq 4 \quad 6 \leq -2B \leq 8$$

$$8 \leq a - 2B \leq 12$$

$$(B) \left. \begin{array}{l} 4 \leq a^2 \leq 16 \quad (1) \quad 2 \leq a \leq 4 \\ 3 \leq -B \leq 4 \end{array} \right\} \begin{array}{l} (2) \quad 6 \leq -aB \leq 16 \\ \Leftrightarrow 12 \leq -2aB \leq 32 \quad (3) \end{array}$$

$$\text{Дпо } (1) + (3) \quad 16 \leq a^2 - 2aB \leq 48$$

118

$$(a) aB(a+B) = 20 \Leftrightarrow a+B = \frac{20}{a} = 5$$

$$(B) x^2 - 5x + 4 = 0 \Leftrightarrow x^2 - 5x + 4 = 0$$

43

$$(119) (a) a^3 B + a^2 B^2 + a^2 B^2 + a B^3 = aB(a^2 + 2aB + B^2) = -12$$

$$\Leftrightarrow a \cdot B (a+B)^2 = -12 \Leftrightarrow a \cdot B (-1)^2 = -12$$

$$\Leftrightarrow aB = -12$$

$$(B) x^2 + x - 12 = 0$$

$$(120) (a) K - \Lambda = 2a^2 + B^2 + 9 - 2a(3-B)$$

$$= 2a^2 + B^2 + 9 - 6a + 2aB$$

$$= (a^2 + 2aB + B^2) + (a^2 - 6a + 9)$$

$$(B) K > \Lambda \Leftrightarrow K - \Lambda > 0 \Leftrightarrow (a+B)^2 + (a-3)^2 > 0 \text{ ισχύει}$$

$$(C) K = \Lambda \Leftrightarrow (a+B)^2 + (a-3)^2 = 0 \Leftrightarrow a = -B \text{ και } a = 3$$

$$\Leftrightarrow a = 3 \text{ και } B = -3$$

$$(121) (a) a^2 B + B = a B^2 + a$$

$$\Leftrightarrow a^2 B - a B^2 + B - a = 0 \Leftrightarrow aB(a-B) - (a-B) = 0$$

$$\Leftrightarrow (a-B)(aB-1) = 0 \Leftrightarrow a=B \text{ ή } aB=1$$

Όπως $a \neq B$ άρα $aB=1$ άρα αντίστροφοι

$$(B) K = \frac{a^{22} \cdot B^{24}}{a^{-2} \cdot a^{25} \cdot B^{25}} = \frac{a^{22} \cdot B^{24}}{a^{23} \cdot B^{25}} = \frac{1}{a \cdot B} = \frac{1}{1} = 1$$

122

$$N(\Omega) = 400, N(A) = 200, P(\Gamma) = \frac{20}{100}$$

$$(a) P(\Gamma) = \frac{N(\Gamma)}{N(\Omega)} \Rightarrow \frac{20}{100} = \frac{N(\Gamma)}{400}$$

$$\Rightarrow \cancel{100} N(\Gamma) = 20 \cdot 400 \Rightarrow \boxed{N(\Gamma) = 80}$$

$$(b) N(\Omega) = N(A) + N(B) + N(\Gamma)$$

$$\Rightarrow N(B) = 400 - 200 - 80$$

$$\Rightarrow N(B) = 120$$

$$(y) P(B) = \frac{N(B)}{N(\Omega)} = \frac{120}{400} = \frac{3}{10}$$

123

$$(a) |2x-3| = 3-2x$$

$$|2x-3| = \begin{cases} 2x-3, & \text{αν } x > \frac{3}{2} \\ 3-2x, & \text{αν } x \leq \frac{3}{2} \end{cases} \quad \text{Ορισμός}$$

$$(b) x \leq \frac{3}{2} \Rightarrow -x \geq -\frac{3}{2} \Rightarrow 3-x \geq \frac{3}{1} - \frac{3}{2}$$

$$\Rightarrow 3-x \geq \frac{3}{2}$$

$$\text{αρα } K = 3 - 2x - 2(3-x) = 3 - \cancel{2x} - 6 + \cancel{2x} = -3$$

$$\textcircled{124} \text{ (a) } (2-x)^2 = (x+4)(6-x)$$

$$\Leftrightarrow 4 - \cancel{4x} + x^2 = 6x - x^2 + 24 - \cancel{4x}$$

$$\Leftrightarrow 2x^2 - 6x - 20 = 0 \Leftrightarrow x^2 - 3x - 10 = 0$$

$$x = 5 \vee x = -2$$

$$\text{(B) (i) } a_4 = 6 - x \Leftrightarrow a_4 = 1$$

$$9, -3, 1, \lambda = \frac{-3}{9} = -\frac{1}{3}$$

$$\forall x \quad \lambda = -\frac{1}{3}$$

$$\text{(ii) } a_4 = 1 \Leftrightarrow a_1 \cdot \lambda^3 = 1 \Leftrightarrow a_1 \cdot \left(-\frac{1}{3}\right)^3 = 1$$

$$\Leftrightarrow a_1 \left(-\frac{1}{27}\right) = 1 \Leftrightarrow \boxed{a_1 = -27}$$

$$\textcircled{125} \text{ (a) } |x-2| < 3 \Leftrightarrow -3 < x-2 < 3 \Leftrightarrow -1 < x < 5$$

$$\text{(B) } k = \frac{|x+1| + |x-5|}{3} = \frac{\cancel{x+1} + 5 - \cancel{x}}{3} = \frac{6}{3} = 2$$

$$\text{Apoia } x > -1 \Leftrightarrow x+1 > 0 \text{ ca } x < 5 \Leftrightarrow x-5 < 0$$

$$\textcircled{126} \text{ (a) } |y-2| < 1 \Leftrightarrow -1 < y-2 < 1 \Leftrightarrow 1 < y < 3$$

$$\forall x \quad y \in (1, 3)$$

$$\text{(B) } k = \frac{\cancel{y-1} + 3 - \cancel{y}}{2} = \frac{2}{2} = 1$$

$$(127) (a) -5 \leq -x \leq -3 \quad \text{or} \quad -2 \leq y \leq -1$$
$$-7 \leq y - x \leq -4$$

$$(b) 9 \leq x^2 \leq 25 \quad \text{or} \quad 1 \leq y \leq 2 \quad \text{or} \quad 1 \leq y^2 \leq 4$$

$$10 \leq x^2 + y^2 \leq 29$$

$$(128) (a) a_1 = 2 \quad \text{or} \quad a_{25} = a_{12} + 39$$

$$\Rightarrow a_1 + 24w = a_1 + 11w + 39 \quad \Rightarrow 13w = 39 \quad \Rightarrow w = 3$$

$$(b) a_v = 152 \quad \Rightarrow a_1 + (v-1) \cdot w = 152$$

$$\Rightarrow (v-1) \cdot 3 = 150 \quad \Rightarrow v-1 = 50 \quad \Rightarrow \boxed{v=51}$$

$$(129) (a) \frac{a_{15} - a_9}{a_{10} - a_7} = \frac{a_1 + 14w - a_1 - 8w}{a_1 + 9w - a_1 - 6w} = \frac{6w}{3w} = 2$$

$$(b) \frac{a_{15} - a_9}{a_{10} - a_7} = 2 \quad \Rightarrow \frac{18}{9} = \frac{2}{1}$$

$$\Rightarrow a_{10} - a_7 = 9 \quad \Rightarrow a_1 + 9w - a_1 - 6w = 9$$

$$\Rightarrow 3w = 9 \quad \Rightarrow \boxed{w=3}$$

$$130 \quad (a+3)x = a^2 - 9$$

$$(a) \quad (i) \quad a=1 \quad \tau \acute{o} \tau \epsilon \quad (1+3)x = 1-9$$

$$\Leftrightarrow 4x = -8 \Leftrightarrow x = -2$$

$$(ii) \quad a = -3 \quad \tau \acute{o} \tau \epsilon \quad (-3+3) \cdot x = (-3)^2 - 9$$

$$\Leftrightarrow 0x = 0 \quad \tau \alpha \upsilon \tau \acute{o} \tau \eta \mu \alpha$$

$$(B) \quad (a+3) \cdot x = (a-3)(a+3) \quad \gamma \acute{i} \alpha \quad \alpha \neq -3$$

$$\boxed{x = a-3}$$

$$131 \quad (a) \quad a_4 - a_9 = 15 \Leftrightarrow a_1 + 3w - a_1 - 8w = 15$$

$$\Leftrightarrow -5w = 15 \Leftrightarrow w = -3$$

$$(B) \quad v \in \mathbb{N}, \quad a_v = v \Leftrightarrow a_1 + (v-1) \cdot w = v$$

$$\Leftrightarrow 41 + (v-1) \cdot (-3) = v$$

$$\Leftrightarrow 41 - 3v + 3 = v \Leftrightarrow 4v = 44 \Leftrightarrow \boxed{v = 11}$$

$$132 \quad (a) \quad a_6 + a_{11} = 40 \Leftrightarrow a_1 + 5w + a_1 + 10w = 40$$

$$\Leftrightarrow 2a_1 + 15w = 40 \Leftrightarrow 2a_1 = 40 - 15 \cdot 4$$

$$\Leftrightarrow 2a_1 = -20 \Leftrightarrow a_1 = -10$$

$$(B) \quad S_v = 0 \Leftrightarrow \frac{v}{2} [2a_1 + (v-1) \cdot w] = 0$$

$$\Leftrightarrow \frac{v}{2} [-20 + 4(v-1)] = 0 \Leftrightarrow v[-10 + 2(v-1)] = 0$$

$$\Leftrightarrow -10 + 2(v-1) = 0 \Leftrightarrow 2(v-1) = 10$$

$$\Leftrightarrow v-1 = 5 \Leftrightarrow \boxed{v=6}$$

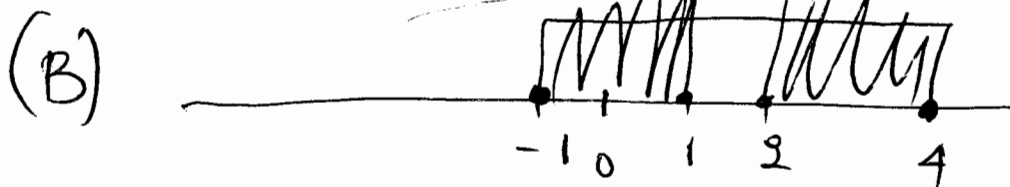
133

$$(a) (i) |2x-3| \leq 5 \Leftrightarrow -5 \leq 2x-3 \leq 5$$

$$\Leftrightarrow -2 \leq 2x \leq 8 \Leftrightarrow -1 \leq x \leq 4$$

$$(ii) |2x-3| \geq 1 \Leftrightarrow 2x-3 \geq 1 \vee 2x-3 \leq -1$$

$$\Leftrightarrow 2x \geq 4 \vee 2x \leq 2 \Leftrightarrow x \geq 2 \vee x \leq 1$$



$$x \in [-1, 1] \cup [2, 4]$$

134

$$(a) 2x^2 - x - 6 = 0 \quad \Delta = 1 - 4 \cdot 2 \cdot (-6) = 1 + 48$$

$$x_{1,2} = \frac{1 \pm 7}{4} \begin{cases} 2 \\ -\frac{3}{2} \end{cases}$$

$$(B) |x-1| < 2 \Leftrightarrow -2 < x-1 < 2 \Leftrightarrow -1 < x < 3$$

$$2 \in (-1, 3), \quad -\frac{3}{2} \notin (-1, 3)$$

135

$$(a) \text{По условию } x^2 - x \neq 0 \text{ или } 1-x \neq 0$$

$$\Leftrightarrow x(x-1) \neq 0 \text{ или } 1 \neq x \Leftrightarrow x \neq 0 \text{ или } x \neq 1$$

$$(B) \frac{2x^2-1}{x(x-1)} - \frac{1}{x-1} = 0 \Leftrightarrow 2x^2-1-x=0$$

$$2x^2 - x - 1 = 0 \quad \Delta = 1 - 4 \cdot 2 \cdot (-1) = 1 + 8 = 3^2$$

(50)

$$x_{1,2} = \frac{1 \pm 3}{4} \begin{cases} 1 \text{ dropp.} \\ -\frac{1}{2} \end{cases} \quad \text{d.p.a. } x = -\frac{1}{2}$$

136 (a) $\Pi = 2x_1 + 2x_2 = 20 \Leftrightarrow x_1 + x_2 = 10$ kai $x_1 \cdot x_2 = 24$
 $x^2 - 10x + 24 = 0$

(B) $x_1 = 4$ kai $x_2 = 6$

137 (a) $(a+b)^2 = a^2 + b^2 + 2ab$

$\Leftrightarrow 12^2 = 272 + 2ab \Leftrightarrow 2ab = 144 - 272$

$\Leftrightarrow 2ab = -128 \Leftrightarrow ab = -64$

(B) $x^2 - 12x - 64 = 0$

(d) $x_1 = 16, x_2 = -4$

138 $A = \sqrt{(x-2)^2}$

(a) Πεπει $(x-2)^2 \geq 0$ d.p.a. για κάθε x

(B) Πεπει $2-x \geq 0 \Leftrightarrow x \leq 2$

(d) $A = \sqrt{(x-2)^2} = |x-2| = 2-x$ d.p.a. $x \leq 2$

$B = \sqrt[3]{(2-x)^3} = 2-x$ d.p.a. $x \leq 2$

(139) (a) $5x+2 = \frac{x+6+11x-6}{2} \Rightarrow 5x+2 = 6x$
 $\Rightarrow x=2$

8, 12, 16 $w = 12 - 8 = 4$

(B) $S_8 = \frac{8}{2} [2 \cdot 0 + 7 \cdot 4] = 4 \cdot (0 + 28) = 112$

(140) (a) $A+B = \frac{1}{3-\sqrt{7}} + \frac{1}{3+\sqrt{7}} = \frac{3+\sqrt{7} + 3-\sqrt{7}}{9-7}$

$\Rightarrow A+B = \frac{6}{2} = 3$

$A \cdot B = \frac{1}{(3-\sqrt{7})(3+\sqrt{7})} = \frac{1}{9-7} = \frac{1}{2}$

(B) $x^2 - 3x + \frac{1}{2} = 0 \Rightarrow 2x^2 - 6x + 1 = 0$

(141) (a) $A \cdot B \cdot C = \sqrt[3]{5} \cdot \sqrt{3} \cdot \sqrt[6]{5} = 5^{\frac{1}{3}} \cdot 5^{\frac{1}{6}} \cdot \sqrt{3}$
 $= 5^{\frac{2}{6} + \frac{1}{6}} \cdot \sqrt{3} = 5^{\frac{3}{6}} \cdot \sqrt{3} = 5^{\frac{1}{2}} \cdot \sqrt{3} = \sqrt{5} \cdot \sqrt{3}$
 $= \sqrt{15}$

(B) $A^6 = (\sqrt[3]{5})^6 = \left[(\sqrt[3]{5})^3 \right]^2 = 25$

$B^6 = (\sqrt{3})^6 = \left[(\sqrt{3})^2 \right]^3 = 3^3 = 27$

$\therefore B^6 > A^6 \Rightarrow B > A$

(52)

$$5 \xrightarrow{\cdot 3} 15 \xrightarrow{\cdot 3} 45 \xrightarrow{\cdot 3} 135$$

$$(142) \quad (a) \quad \frac{a_5}{a_2} = 27 \Leftrightarrow \frac{a_1 \cdot \lambda^4}{a_1 \cdot \lambda} = 27$$

$$\Leftrightarrow \lambda^3 = 27 \Leftrightarrow \lambda = \sqrt[3]{27} \Leftrightarrow \boxed{\lambda = 3}$$

$$(B) \quad S_4 = 200 \Leftrightarrow a_1 \frac{\lambda^4 - 1}{\lambda - 1} = 200$$

$$\Leftrightarrow a_1 \cdot \frac{3^4 - 1}{3 - 1} = 200 \Leftrightarrow a_1 \cdot \frac{80}{2} = 200$$

$$\Leftrightarrow 40 \cdot a_1 = 200 \Leftrightarrow a_1 = \frac{200}{40} \Leftrightarrow a_1 = 5$$

$$(143) \quad (a) \quad A \cdot B = (2 - \sqrt{3})(2 + \sqrt{3}) = 4 - 3 = 1$$

$$(B) \quad \Pi = A^2 + B^2 = (A+B)^2 - 2A \cdot B = 4^2 - 2 \cdot 1 = 16 - 2 = 14$$

$$A+B = 2 - \sqrt{3} + 2 + \sqrt{3} = 4$$

$$(144) \quad (a) \quad a = \lambda + 2, \quad B = 2\lambda, \quad \gamma = \lambda - 1$$

$$\Delta > 0 \Leftrightarrow 4\lambda^2 - 4(\lambda + 2)(\lambda - 1) > 0$$

$$\Leftrightarrow 4\lambda^2 - 4(\lambda^2 - \lambda + 2\lambda - 2) > 0 \quad \cancel{4\lambda^2} - \cancel{4\lambda^2} - 4\lambda + 8 > 0$$

$$\Leftrightarrow 4\lambda < 8 \Leftrightarrow \boxed{\lambda < 2}$$

$$(B) \quad x_1 \cdot x_2 = \frac{\gamma}{a} = \frac{\lambda - 1}{\lambda + 2} = -3$$

$$\Leftrightarrow \lambda - 1 = -3\lambda - 6 \Leftrightarrow 4\lambda = -5 \Leftrightarrow \boxed{\lambda = -\frac{5}{4}}$$

145

(a) $|2x-1| < 1 \Leftrightarrow -1 < 2x-1 < 1$

$\Leftrightarrow 0 < 2x < 2 \Leftrightarrow 0 < x < 1$

(B) $x-x^2 = x(x-1) < 0$

$x < 1 \wedge x-1 < 0$

$\Leftrightarrow x-x^2 < 0 \Leftrightarrow x < x^2$

$\Leftrightarrow x < x^2 < 1 \wedge 0 < x < 1 \Leftrightarrow 0 < x^2 < 1$

146

(a) $a_5 = 14 \Leftrightarrow a_1 + 4w = 14 \Leftrightarrow 4w = 12 \Leftrightarrow w = 3$

(B) $S_v = 77 \Leftrightarrow \frac{v}{2} [2 \cdot a_1 + (v-1) \cdot w] = 77$

$\Leftrightarrow v [2 \cdot 2 + (v-1) \cdot 3] = 2 \cdot 77$

$\Leftrightarrow v (4 + 3v - 3) = 2 \cdot 77 \Leftrightarrow (3v+1)v = 154$

$\Leftrightarrow 3v^2 + v - 154 = 0 \quad \Delta = 1 + 12 \cdot 154 = 1849$

$v_{1,2} = \frac{-1 \pm 43}{6} \begin{cases} 7 \\ -\frac{99}{3} \end{cases}$ $\Leftrightarrow v = 7$ $\wedge v \in \mathbb{N}^*$

147

(a) $\Delta = k^2 - 4 \cdot 1 \cdot (-2) = k^2 + 8 \geq 0$

(B) (i) $S = x_1 + x_2 = -\frac{B}{a} = -\frac{(-3)}{1} = 3$

$P = x_1 \cdot x_2 = \frac{\gamma}{a} = \frac{-2}{1} = -2$

(ii) $S' = P_1 + P_2 = 2x_1 + 2x_2 = 2(x_1 + x_2) = 2 \cdot 3 = 6$

$P' = P_1 \cdot P_2 = 4x_1 \cdot x_2 = 4 \cdot (-2) = -8$

$$x^2 - 5x + 6 = 0 \Leftrightarrow x^2 - 6x - 8 = 0 \quad (54)$$

$$(148) \quad (a) \quad a + \frac{4}{a} \geq 4 \quad (a > 0) \Leftrightarrow a^2 + 4 \geq 4a$$

$$\Leftrightarrow a^2 - 4a + 4 \geq 0 \Leftrightarrow (a-2)^2 \geq 0 \quad \text{το ίδιο}$$

$$(B) \quad a + \frac{4}{a} \geq 4 \quad (1) \quad \text{και} \quad b + \frac{4}{b} \geq 4 \quad (2)$$

Ολοκληρωμένα τα a και b είναι θετικά

$$\text{από (1) \cdot (2)} \quad \left(a + \frac{4}{a}\right) \left(b + \frac{4}{b}\right) \geq 16$$

$$(149) \quad (a) \quad K - \Lambda \geq 0 \Leftrightarrow 2a^2 + b^2 - 2ab \geq 0$$

$$\Leftrightarrow a^2 + (a-b)^2 \geq 0 \quad \text{το ίδιο}$$

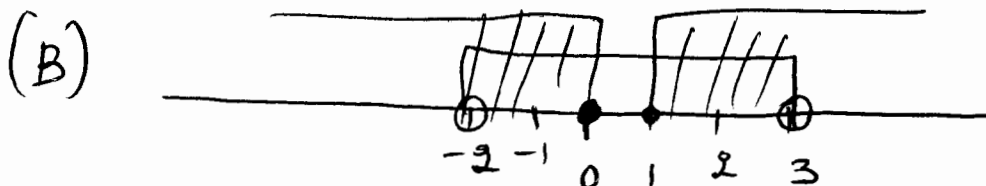
$$(B) \quad \text{Πρέπει} \quad a^2 + (a-b)^2 = 0 \Leftrightarrow a = 0 \quad \text{και} \quad a = b = 0$$

$$(150) \quad (a) \quad |1 - 2x| < 5 \Leftrightarrow -5 < 1 - 2x < 5$$

$$\Leftrightarrow -6 < -2x < 4 \Leftrightarrow \underline{-2 < x < 3}$$

$$|1 - 2x| \geq 1 \Leftrightarrow 1 - 2x \geq 1 \quad \vee \quad 1 - 2x \leq -1$$

$$\Leftrightarrow 2x \leq 0 \quad \vee \quad 2x \geq 2 \Leftrightarrow \underline{x \leq 0} \quad \vee \quad \underline{x \geq 1}$$



$$x = -1, 0, 1, 2$$

